

# Магнитная левитация в космических объектах

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Scientific Correspondence

*Nature* **400**, 323-324 (22 July 1999) | doi:10.1038/22444

## Magnet levitation at your fingertips

A. K. Geim<sup>1</sup>, M. D. Simon<sup>2</sup>, M. I. Boamfa<sup>1</sup> & L. O. Heflinger<sup>2</sup>

### Abstract

**The stable levitation of magnets is forbidden by Earnshaw's theorem...**

**Nevertheless, levitation of a magnet without using superconductors is widely thought to be impossible. We find that the stable levitation of a magnet can be achieved using the feeble diamagnetism of materials that are normally perceived as being non-magnetic, so that even human fingers can keep a magnet hovering in mid-air without touching it.**

### References

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# Магнитная проницаемость $\vec{B} = \mu\vec{H}$

Самуэль Ирншоу

1842 г. “Проблема свободного подвеса”

Статическая система  $F \propto r^{-2}$  неустойчива

Майкл Фарадей

1846 г. Диамagnetизм

Уильям Томсон (Лорд Кельвин)

1872 г. Проблема “Гроба Магомета”:

возможно, но сомнительно...

Вернер Браунбек

1939 г. левитация

(диамagnetики и сверхпроводники)

графит (1см, 75мг) в поле  $B = 20000$  Гс

Владимир Константинович Аркадьев

1945 г. левитация шара

магнит над сверхпроводящим диском

## МАГНЕТИЗМ

Магнитные свойства вещества

### МАГНЕТИКИ

СЛАБОМАГНИТНЫЕ  
ВЕЩЕСТВА

СИЛЬНОМАГНИТНЫЕ  
ВЕЩЕСТВА

ДИАМАГНЕТИКИ

ПАРАМАГНЕТИКИ

ФЕРРОМАГНЕТИКИ

Водород

Бензол

Вода

Медь

Стекло

Кварц

Каменная соль

Висмут

Графит

Азот

Воздух

Кислород

Эбонит

Алюминий

Вольфрам

Платина

Железо

Никель

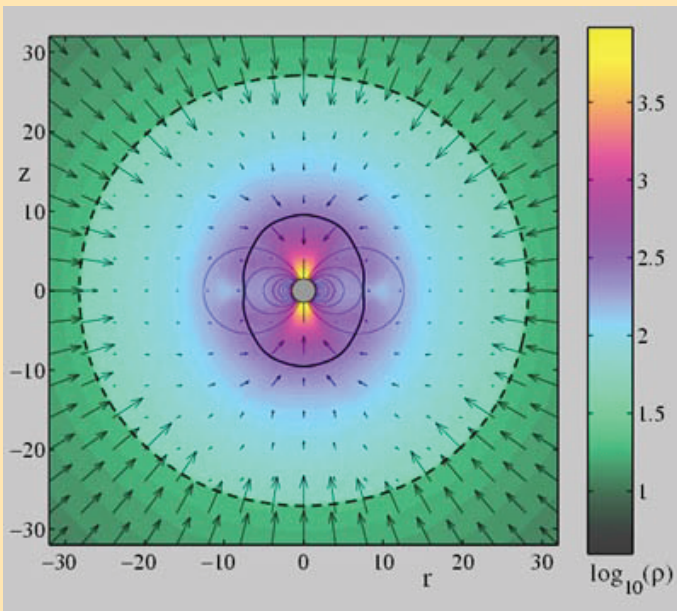
Кобальт

$$\mu \leq 1$$

$$\mu \geq 1$$

$$\mu \gg 1$$

$\mu$  - магнитная проницаемость вещества



## Quasi-spherical accretion

$$\frac{\mathcal{E}_{\text{ram}}(r)}{dr} < \frac{\mathcal{E}_{\phi}(r)}{dr} < \frac{\mathcal{E}_{\text{m}}(r)}{dr}$$

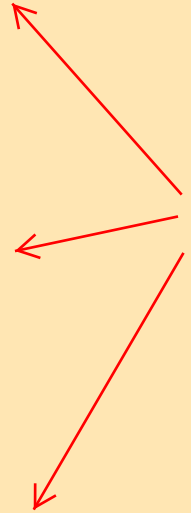
- Mass conservation  $\mathcal{E}_{\text{ram}}(r) \sim \rho(r) v_{\text{ff}}^2(r) \propto \left(\frac{r_{\text{G}}}{r}\right)^{5/2}$

- Angular momentum conservation

$$\mathcal{E}_{\phi}(r) \sim \rho(r) v_{\phi}^2(r) \propto \left(\frac{r_{\text{G}}}{r}\right)^{7/2}$$

- Magnetic flux conservation

$$\mathcal{E}_{\text{m}}(r) \sim \rho(r) v_{\text{A}}^2(r) \propto \left(\frac{r_{\text{G}}}{r}\right)^4$$



$$v_A = \frac{B}{\sqrt{4\pi\rho}}$$

## BASIC SCALES

$$\beta = \frac{8\pi n k_B T}{B^2}$$

---

- Alfvén radius  $\mu^2/2\pi r_A^6 = \mathcal{E}_{\text{ram}}(r_A)$

$$r_A = \left( \frac{\mu^2}{\dot{m} (2GM_{\text{ns}})^{1/2}} \right)^{2/7}$$

- Circularization radius  $\mathcal{E}_\phi(r_{\text{circ}}) = \mathcal{E}_{\text{ram}}(r_{\text{circ}})$

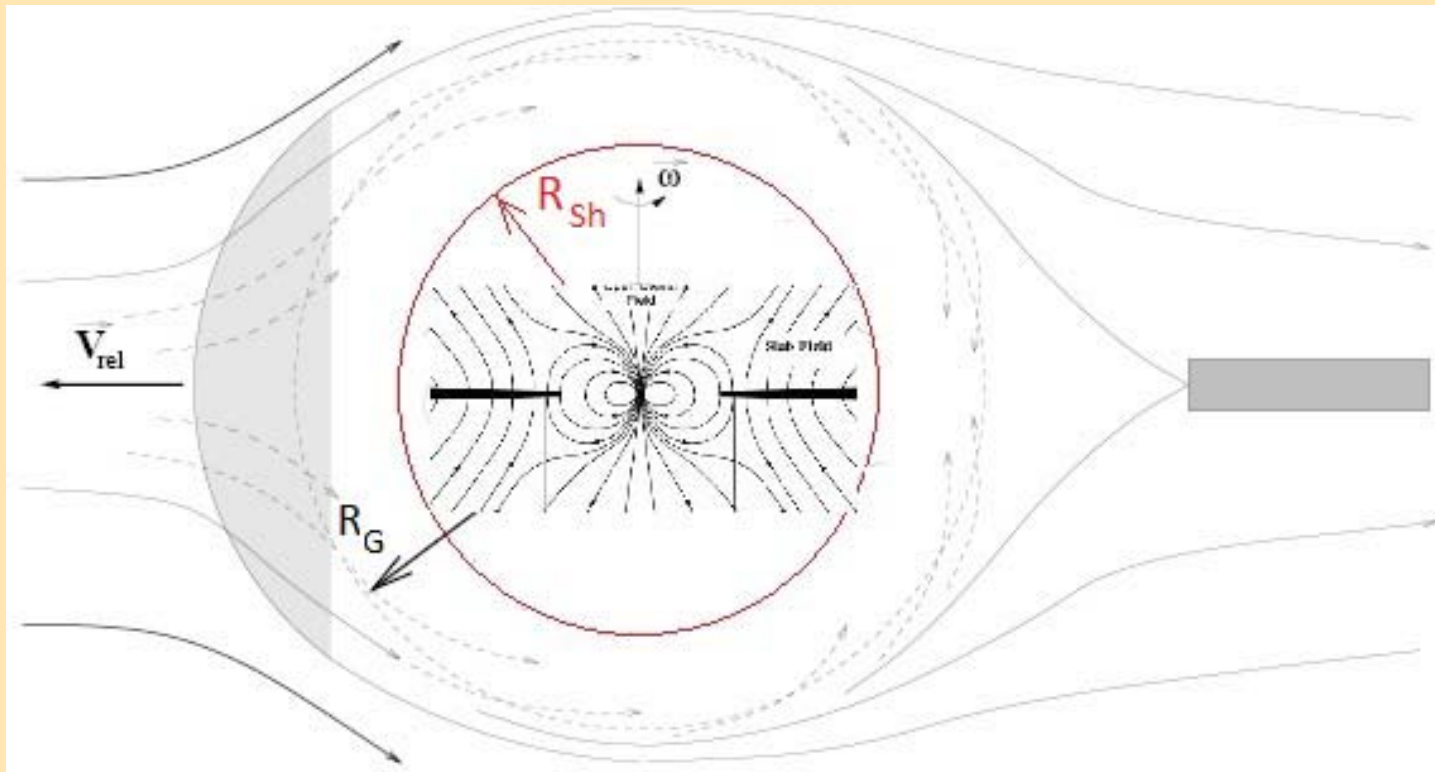
$$r_{\text{circ}} = \frac{\xi^2 \Omega_{\text{orb}}^2 r_G^4}{GM_{\text{ns}}}$$

- Magnetic Levitation radius  $\mathcal{E}_m(R_{\text{sh}}) = \mathcal{E}_{\text{ram}}(R_{\text{sh}})$

$$R_{\text{sh}} = \beta^{-2/3}(r_G) \left( \frac{c_s(r_G)}{v_{\text{rel}}} \right)^{4/3} r_G \sim \left( \frac{v_A(r_G)}{v_{\text{rel}}} \right)^{4/3} r_G$$

# Magnetic Levitation Accretion

$$R_{\text{sh}} > \max\{r_A, r_{\text{circ}}\}$$



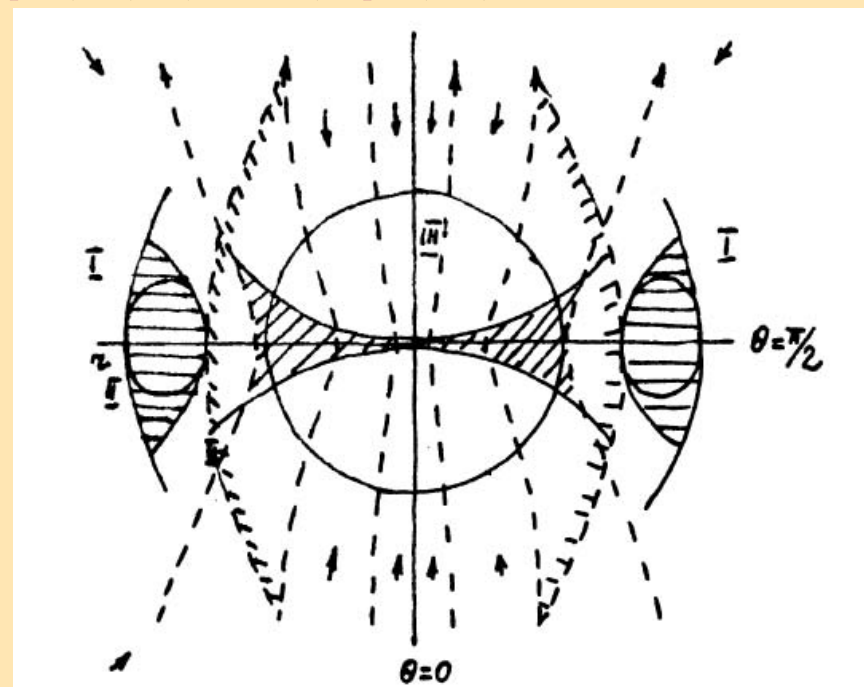
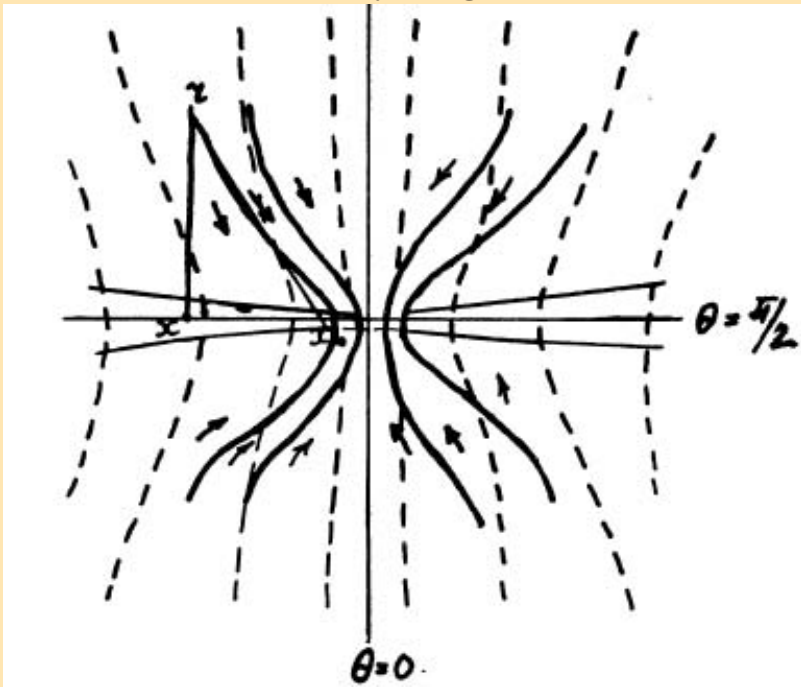
## Non-Keplerian Magnetic self-Levitating Disk

$$R_{\text{sh}} = \beta_0^{-2/3} \left( \frac{c_s(r_G)}{v_{\text{rel}}} \right)^{4/3} r_G$$

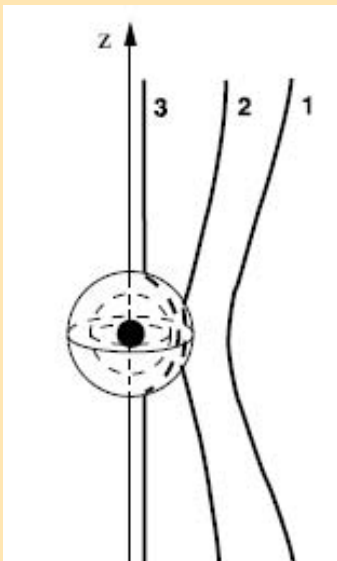
# Non-Keplerian Magnetically-Levitating Disk

$$t_{\text{rec}} = \frac{r}{\eta_m v_A} = \eta_m^{-1} t_{\text{ff}} \left( \frac{v_{\text{ff}}}{v_A} \right)$$

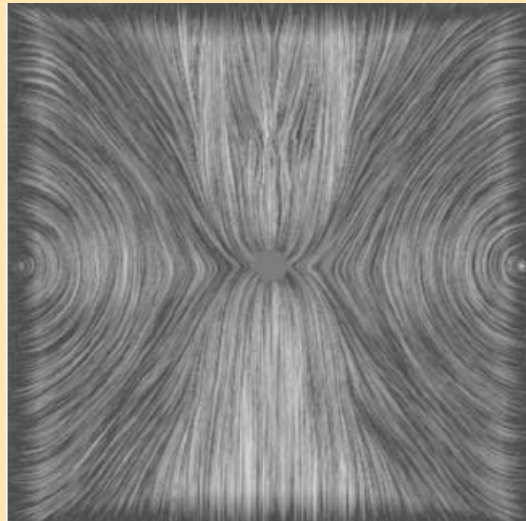
Bisnovaty-Kogan & Ruzmaikin 1974, ApSS, 28, 45; 1976, ApSS, 42, 401



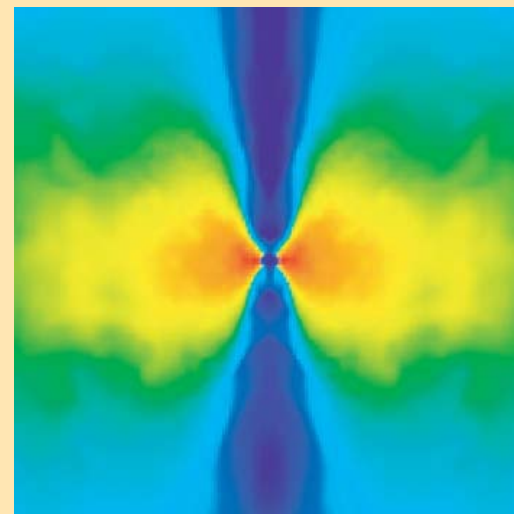
Igumenshev, Narayan & Abramowicz 2003, ApJ, 592, 1042



Magnetic field



Density



To summarize the situation for a highly evolved spiral galaxy like ours (with only a few per cent of the mass remaining in the form of gas): Objects of the order of  $10^6 M_{\odot}$ , more than typical globular cluster masses, are required to initiate a catastrophic accretion process. Unlike theories involving the gravitational collapse of *isolated* objects, the time scale of our process never becomes very short due to the self-limiting effects of radiation pressure. In our Galaxy, in fact, dust grains would keep the time scale well above  $10^8$  years, which is much longer than required for quasi-stellar objects, and the luminosity too low ( $< 10^{12} L_{\odot}$ ). The situation is likely to be more favorable in systems less evolved than our Galaxy where (a) a larger fraction of the mass is still in the form of gas and (b) the relative abundance of heavier elements, and hence of dust grains, is lower. This leads to a shorter time scale and a larger limiting value for the luminosity-mass ratio for the accreting condensation.

I am indebted to Drs. R. P. Feynman, M. Ruderman, M. Schwarzschild, E. Spiegel, L. Spitzer, and L. Woltjer for helpful criticism and suggestions. In fact, they have contributed most of the positive ideas in this note without being responsible for any of the unwarranted conjectures. I am also grateful to the National Academy of Sciences for a senior postdoctoral fellowship.

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В качестве падающего материала можно представить себе вещество второй звезды при коллапсе первой звезды близкой пары. Это может быть та часть оболочки самой коллапсирующей звезды, которая была выброшена перед самым моментом гравитационного самозамыкания: наряду с веществом, приобретающим гиперболическую скорость, часть выбрасываемого вещества может оказаться в запасе на далеких, но замкнутых орбитах.

В самом общем виде идея о падении в мощном гравитационном поле как источнике энергии радиоизлучения высказывалась И. С. Шкловским<sup>(13)</sup>.

Пользуюсь случаем выразить искреннюю благодарность И. Д. Новикову и И. С. Шкловскому за многочисленные дискуссии.

Поступило  
6 XII 1963

### ЦИТИРОВАННАЯ ЛИТЕРАТУРА

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И. С. ШКЛОВСКИЙ

## О ПРИРОДЕ РАДИОГАЛАКТИК

На основании анализа всех существующих наблюдательных данных для радиогалактик строится диаграмма «абсолютная радиовеличина — линейные размеры». На этой диаграмме намечаются две последовательности. «Главная последовательность» характеризуется ростом радиосветимости по мере увеличения размеров излучающей области и совпадением последней с оптически наблюдаемой галактикой. «Последовательность гигантов» характеризуется быстрым уменьшением радиосветимости при увеличении размеров областей радиоизлучения с оптически наблюдаемыми галактиками (например, Лебедь-А, Центавр-А). Для источников, образующих эту последовательность, весьма распространена двойственность.

Обсуждается вопрос о возможных путях эволюции источников, лежащих на последовательности гигантов. В заключение выдвигается новая гипотеза о происхождении радиогалактик. Согласно этой гипотезе, при падении в области ядра массивной сферической галактики межгалактического газа при некоторых условиях там может идти достаточно эффективный процесс ускорения заряженных частиц. При этом ядра некоторых галактик можно рассматривать как циклически работающие ускорители.

# THE ASTROPHYSICAL JOURNAL

## LETTERS TO THE EDITOR

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NUMBER 1, PART 2

ON THE NATURE OF THE SOURCE OF X-RAY EMISSION OF SCO XR-1

The flux of gas in the stream is estimated as  $10^{16}$ – $10^{17}$  gm/sec ( $\sim 10^{-9} M_{\odot}$ /year). When this gas falls on the neutron star the production of energy per unit mass may amount to  $\sim 10^{20}$  ergs/gm. Thus it follows that the suggested modification of the mechanism of the accretion of gas on the neutron star gives the possibility of explaining the power of X-ray emission of the source Sco XR-1.

I. S. SHKLOVSKY

February 3, 1967

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MOSCOW, U.S.S.R.

Так как все вещество стекает в области магнитных полюсов, имея большую кинетическую энергию, то оно может излучать в широком диапазоне частот от радио до рентгена. Это излучение окажется переменным с периодом, равным периоду пульсаций  $\Omega$ . Если ось магнитного диполя не будет совпадать с осью вращения  $\Omega$ , то источник будет переменным с периодом, равным периоду вращения  $\Omega$ .

Шемахинская астрофизическая  
обсерватория АН Азерб.

ССР

июнь, 1969

П.Р. Амнуэль

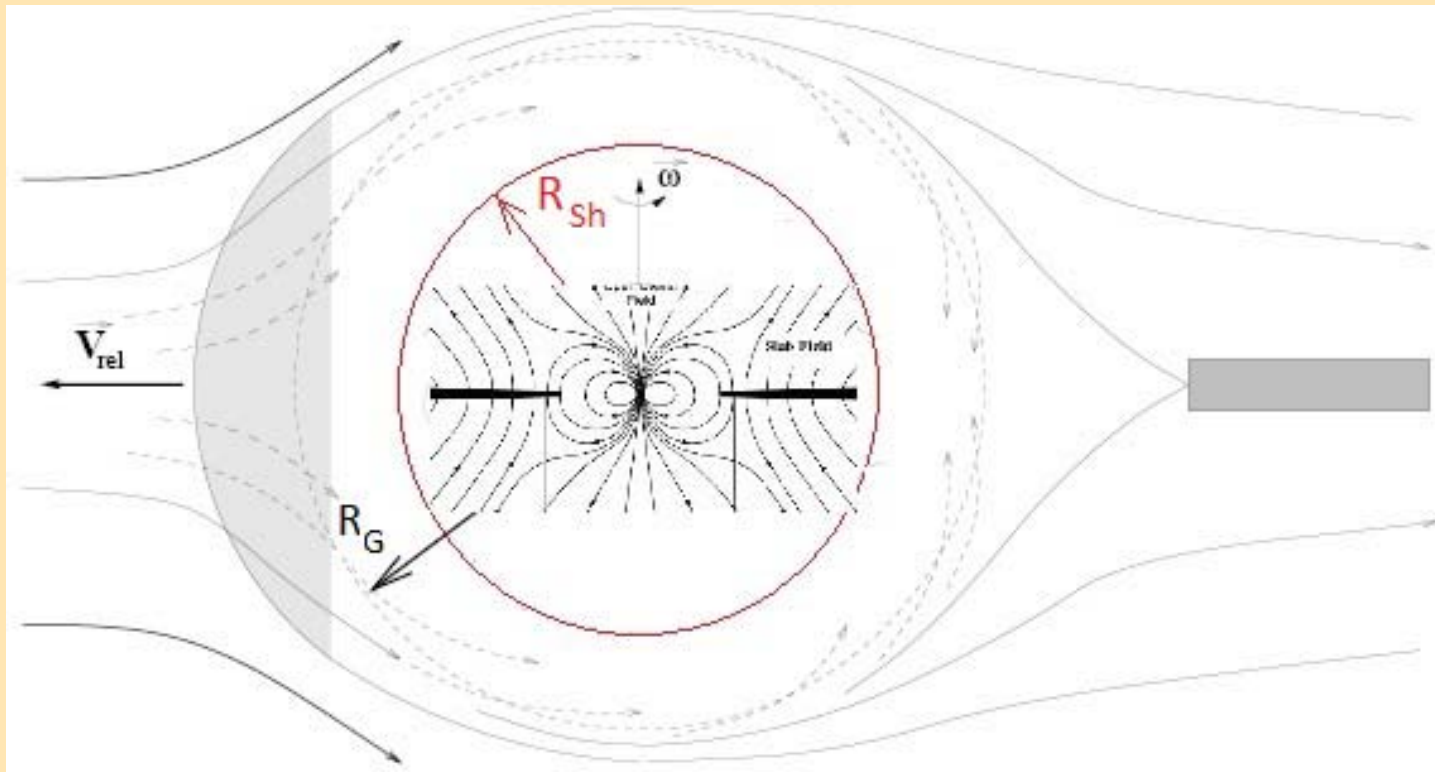
P.R. Amnuel

О.Х. Гусейнов

O.H. Guseinov

# Magnetic Levitation Accretion

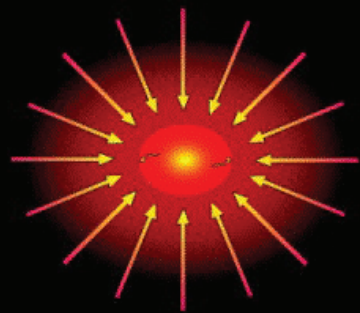
$$R_{\text{sh}} > \max\{r_A, r_{\text{circ}}\}$$



## Non-Keplerian Magnetic self-Levitating Disk

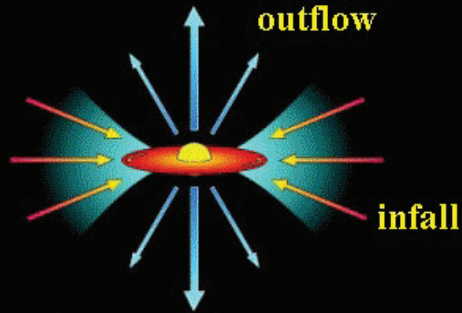
$$R_{\text{sh}} = \beta_0^{-2/3} \left( \frac{c_s(r_G)}{v_{\text{rel}}} \right)^{4/3} r_G$$

# How are single stars born?



**Cloud collapse**

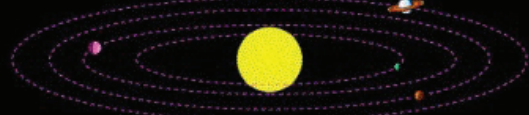
x1000  
in scale



**Rotating disk**



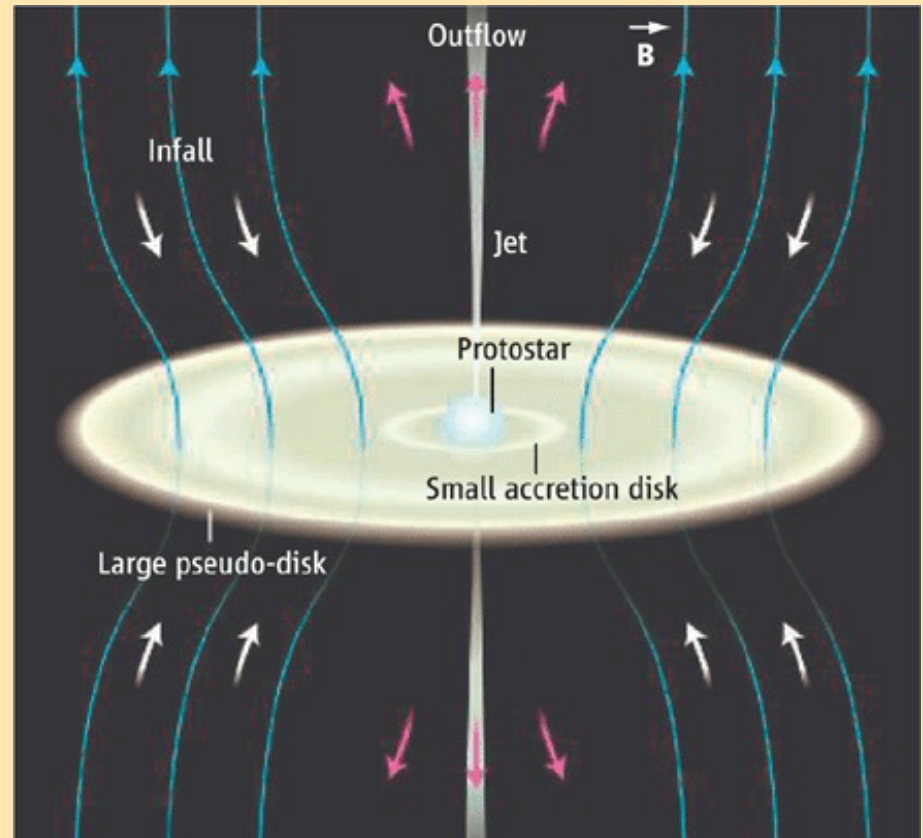
**Planet formation**



**Mature solar system**

Scenario largely from indirect tracers.

Fig. by McCaughrean



# Magnetic Levitation Accretion (MLA) in X-ray Pulsars

Basic condition:

$$R_{\text{sh}} > \max\{r_A, r_{\text{circ}}\} \longrightarrow v_{\text{cr}} < v_{\text{rel}} < v_{\text{ma}}$$

$$v_{\text{cr}} \leq v_{\text{ma}}$$

$$\beta_0 \leq \beta_{\text{max}}$$

$$\beta_{\text{max}} \simeq 164 \times \xi_{0.2}^{-5/4} m^{-1/4} c_6^2 P_{50}^{5/4} \mu_{30}^{-1/2} \dot{m}_{15}^{1/4}$$

## Three possible wind-fed accretion scenarios

**Quasi-spherical**      $v_{\text{rel}} > v_{\text{ma}} \simeq 465 \text{ km s}^{-1} \times \beta_0^{-1/5} \mu_{30}^{-6/35} m^{12/35} \dot{m}_{15}^{3/35} c_6^{2/5}$

**Keplerian disk**      $v_{\text{rel}} < v_{\text{cr}} \simeq 100 \text{ km s}^{-1} \times \beta_0^{1/7} \xi_{0.2}^{3/7} m^{3/7} P_{40}^{-3/7} c_6^{2/5}$

**MAGLEV Disk**      $v_{\text{cr}} < v_{\text{rel}} < v_{\text{ma}}$

# Magnetic Levitation Accretion (MLA) in X-ray Pulsars

Basic condition:  $R_{\text{sh}} > \max\{r_A, r_{\text{circ}}\}$   $v_{\text{ca}} < v_{\text{rel}} < v_{\text{ma}}$

$$v_{\text{ca}} \leq v_{\text{ma}}$$

$$\beta_{\text{max}} \simeq 164 \times \xi_{0.2}^{-5/4} m^{-1/4} c_6^2 P_{50}^{5/4} \mu_{30}^{-1/2} \dot{m}_{15}^{1/4}$$

$$\beta_0 \leq \beta_{\text{max}}$$

Magnetic field of the stellar wind is  $B \geq B_{\text{min}}$

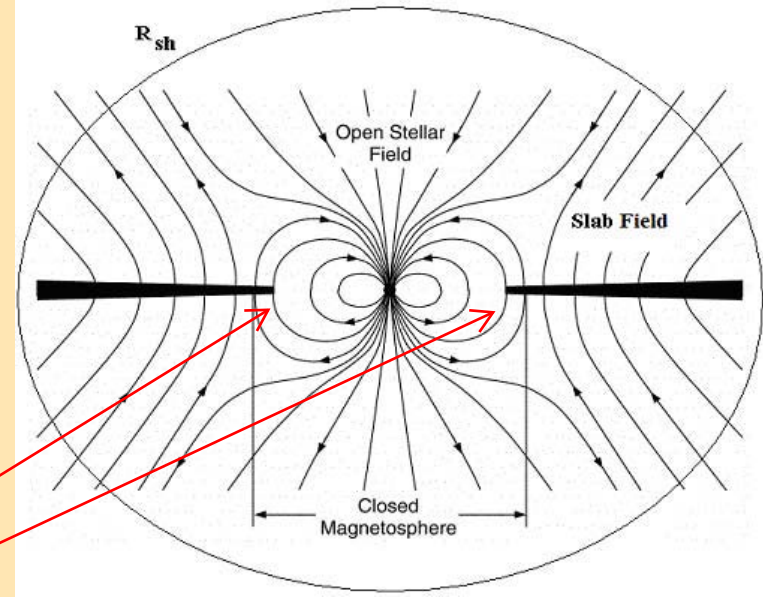
$$B_{\text{min}} = \left( \frac{2 \dot{m} v_{\text{rel}}^3 c_{\text{so}}^2}{(GM_{\text{ns}})^2 \beta_{\text{max}}} \right)^{1/2} \simeq 6 \times 10^{-4} \text{ G} \times$$

$$\times \xi_{0.2}^{5/8} \mu_{30}^{1/4} m^{-7/8} P_{50}^{-5/8} \dot{m}_{15}^{3/8} \left( \frac{v_{\text{rel}}}{100 \text{ km s}^{-1}} \right)^{3/2}$$



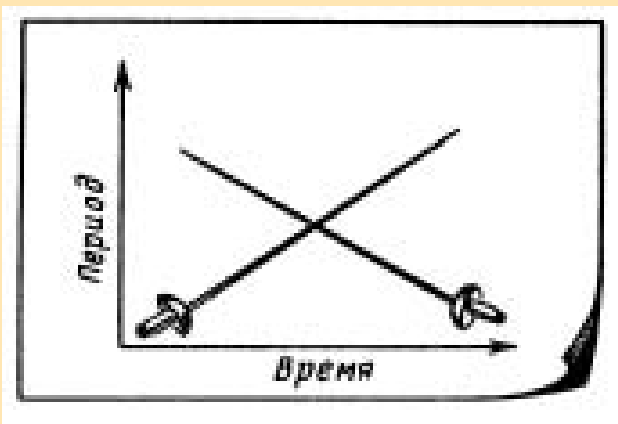
# Magnetically-Levitating (MAGLEV) Disk

$$\left\{ \begin{array}{l} \frac{\mu^2}{2\pi r_{\text{ma}}^6} = \rho(r_{\text{ma}}) c_s^2(r_{\text{ma}}) \\ \dot{\mathcal{M}}_{\text{in}}(r_{\text{ma}}) = \frac{L_X R_{\text{ns}}}{GM_{\text{ns}}} \\ \dot{\mathcal{M}}_{\text{in}}(r_{\text{ma}}) = 4\pi r_{\text{ma}} \delta_m \rho(r_{\text{ma}}) v_{\text{ff}}(r_{\text{ma}}) \\ \delta_m(r_{\text{ma}}) = \left[ t_{\text{ff}}(r_{\text{ma}}) D_{\text{eff}}(r_{\text{ma}}) \right]^{1/2} \end{array} \right.$$



$$D_{\text{eff}}(r_{\text{ma}}) = \alpha D_B(r_{\text{ma}}) = \alpha \frac{ck_B T_i(r_{\text{ma}})}{16eB(r_{\text{ma}})}$$

$$r_{\text{ma}} = \left( \frac{cm_p^2}{16\sqrt{2}ek_B} \right)^{2/13} \frac{\alpha^{2/13} \mu^{6/13} (GM_{\text{ns}})^{5/13}}{T_0^{2/13} L_X^{4/13} R_{\text{ns}}^{4/13}}$$

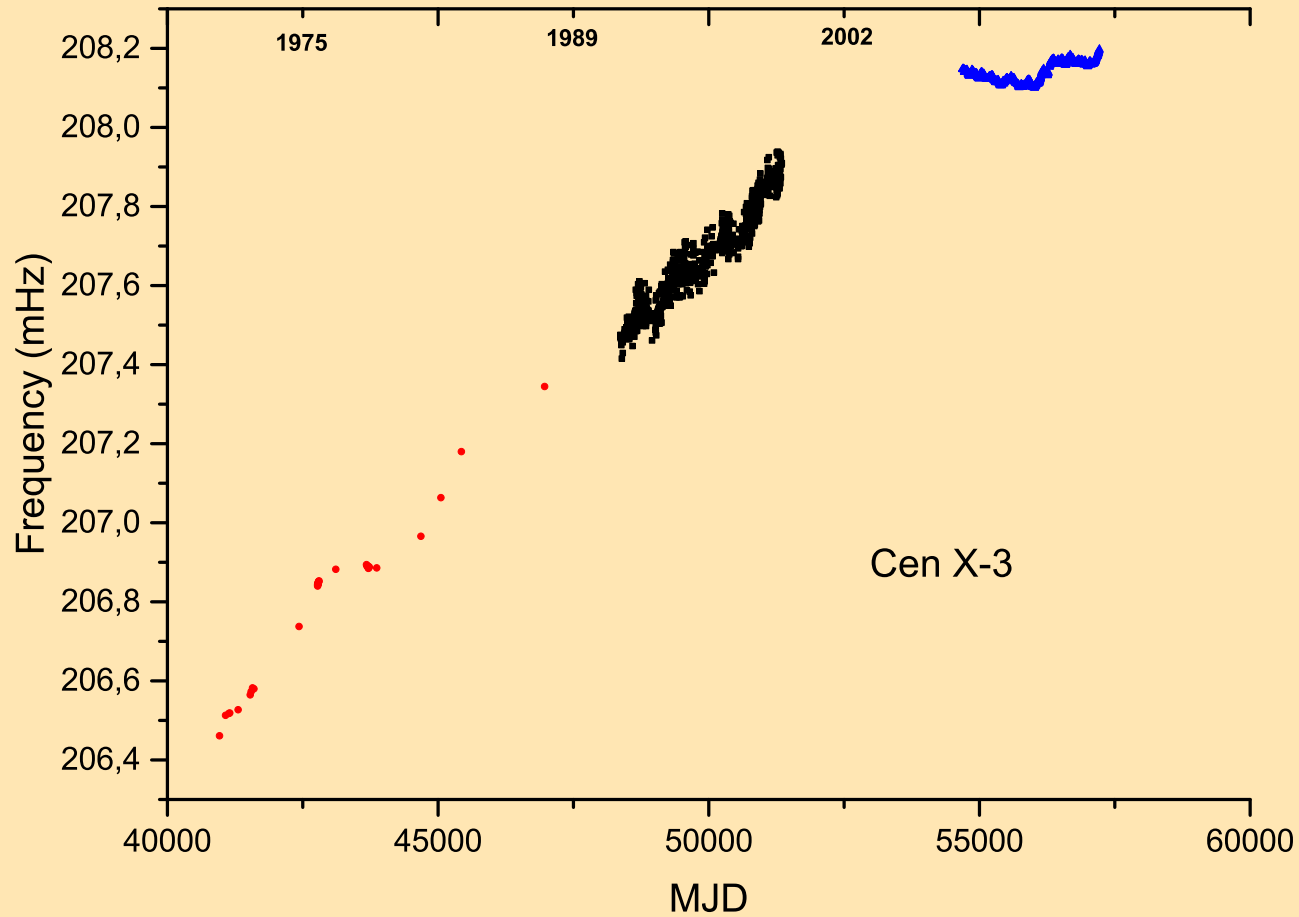


Shvartsman, V.F. “Two generations of pulsars”, 1970

Radiophysics and Quantum Electronics, Vol. 13, Issue 12, pp. 1428-1440

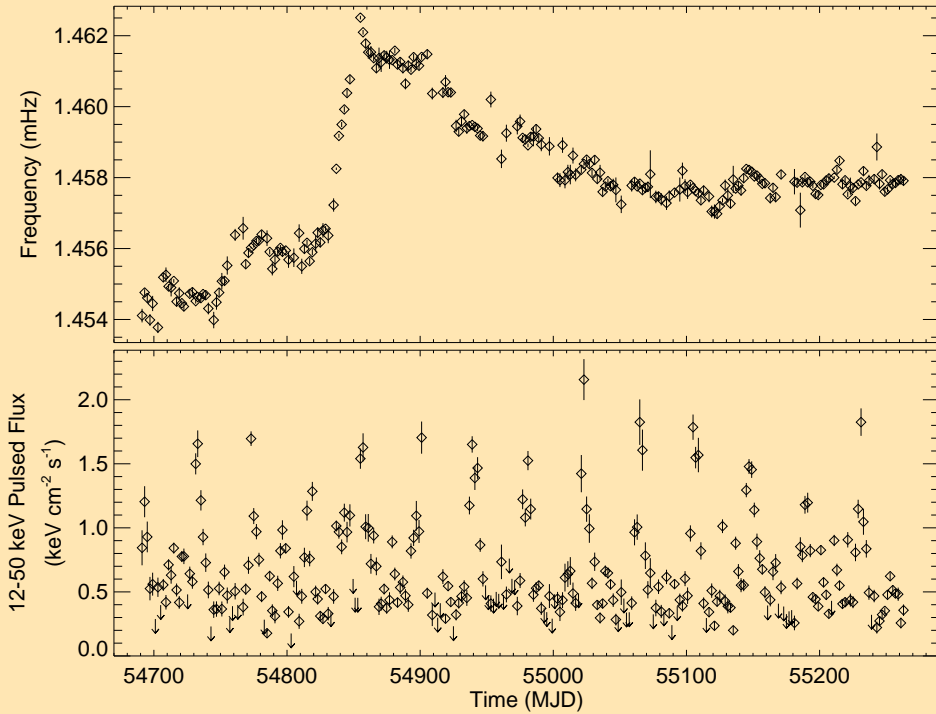
$$\nu = \frac{1}{P_s}$$

$$P_s \simeq 4.8 \text{ s}$$

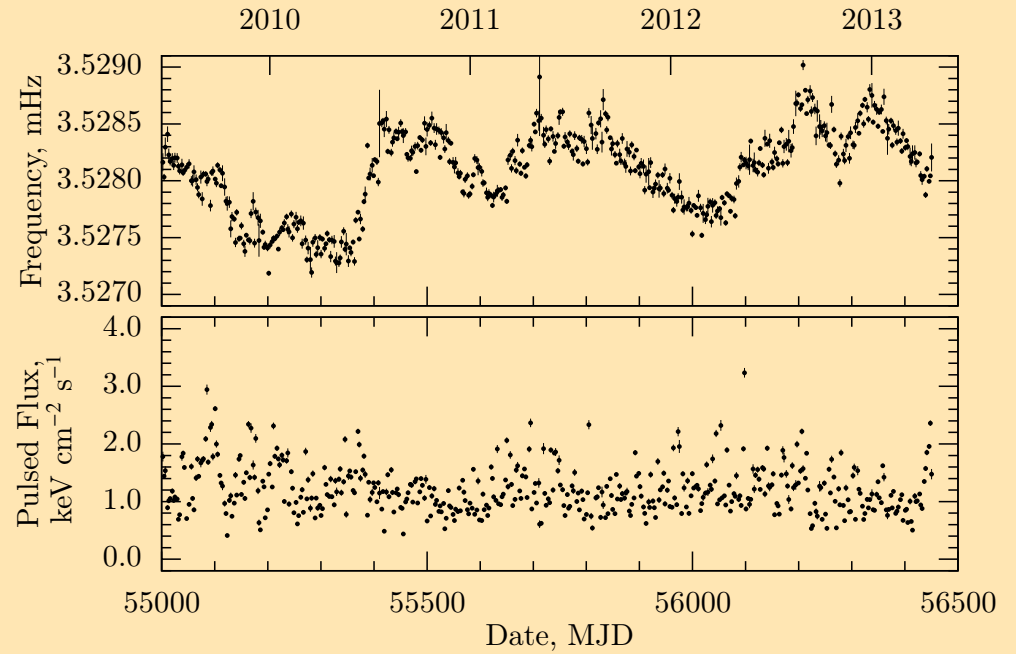


# Spin Evolution of X-ray Pulsars

GX 301-2



Vela X-1



$$|2\pi I \dot{\nu}| = |K_{\text{su}}| - |K_{\text{sd}}|$$

$$|K_{\text{sd}}| \geq 2\pi I |\dot{\nu}_{\text{sd}}|$$

$$|K_{\text{su}}| \geq 2\pi I |\dot{\nu}_{\text{su}}|$$

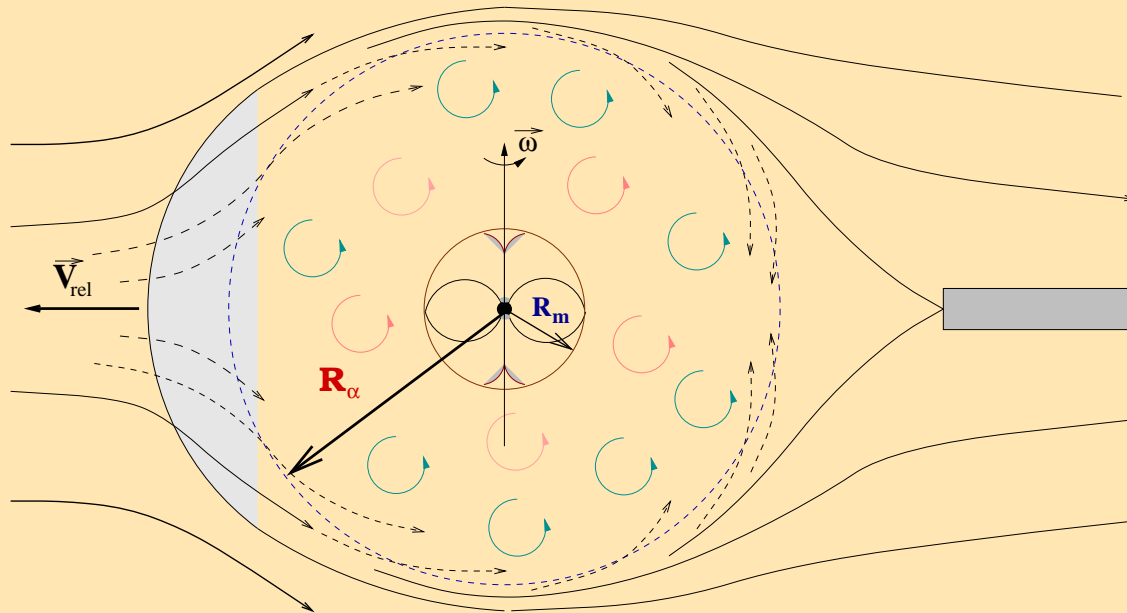
$$P_{\text{eq}} \equiv P_{\text{s}} (|K_{\text{su}}| = |K_{\text{sd}}|)$$

# Torque definition

$$\mathbf{K} = \mathbf{r} \times \mathbf{F}$$

**Model task:**

A sphere of the radius  $r_m$   
is rotating in a viscose medium



# Torque exerted on a NS at its magnetosphere

$$K(r_m) = \nu_t S_{\text{eff}}(r_m) \rho(r_m) v_\phi(r_m)$$

Viscosity

$$\nu_t = k_t \ell_t v_t$$

$$\ell_t \leq r_m$$

Effective Area

$$S_{\text{eff}} = 2\pi r_m \left( \frac{c_s^2(r_m) r_m^2}{GM_{\text{ns}}} \right)$$

$$v_t \leq v_k(r_m)$$

Density

$$\rho(r_m) = \frac{1}{c_s^2(r_m)} \left( \frac{\mu^2}{2\pi r_m^6} \right)$$

$$r_{\text{cor}} = \left( \frac{GM_{\text{ns}}}{\omega_s^2} \right)^{1/3}$$

$\phi$ -velocity

$$v_\phi = r_m [\omega_s - \Omega(r_m)]$$

$$K(r_m) = k_t \frac{\mu^2}{(r_m r_{\text{cor}})^{3/2}} \left( \frac{\Omega(r_m)}{\omega_s} - 1 \right)$$

Ikhsanov, N.R. & Beskrovnaya, N.G. *Astronomy Reports*, 56, 589 (2012)

Ikhsanov, N.R. & Finger, M.H. *Astrophysical Journal*, 753, 1 (2012)

Ikhsanov, N.R., Kim, V.Yu., Beskrovnaya, N.G. & Pustil'nik, L.A. *ApSS*, 346, 105 (2013)

# Correspondence principle is satisfied

$$|K_{\text{sd}}(r_{\text{m}})| \leq \frac{\mu^2}{(r_{\text{m}} r_{\text{cor}})^{3/2}} \left(1 - \frac{\Omega(r_{\text{m}})}{\omega_{\text{s}}}\right)$$

## I. Accretion from a Turbulent-dominated Atmosphere

$$\Omega(r_{\text{m}}) = 0$$

$$r_{\text{m}} \equiv r_{\text{A}} = \left(\frac{\mu^2}{\mathfrak{M} \sqrt{2GM_{\text{ns}}}}\right)^{2/7}$$

$$v_{\text{t}}(r_{\text{A}}) = v_{\text{k}}(r_{\text{A}})$$

$$|K_{\text{sd}}^{(\text{t})}(r_{\text{A}})| \leq \mathfrak{M} \omega_{\text{s}} r_{\text{A}}^2$$

## II. Accretion from a free-falling material (Bondi accretion scenarios)

$$\Omega(r_{\text{m}}) = 0$$

$$r_{\text{m}} \equiv r_{\text{A}}$$

$$v_{\text{t}}(r_{\text{A}}) = \omega_{\text{s}} r_{\text{A}}$$

$$|K_{\text{sd}}^{(\text{ff})}(r_{\text{A}})| = |K_{\text{sd}}^{(\text{t})}(r_{\text{A}})| \times \left(\frac{\omega_{\text{s}} r_{\text{A}}}{v_{\text{k}}}\right) \leq \frac{\mu^2}{r_{\text{cor}}^3}$$

# Evaluation of the Magnetospheric Radius from Spin Evolution

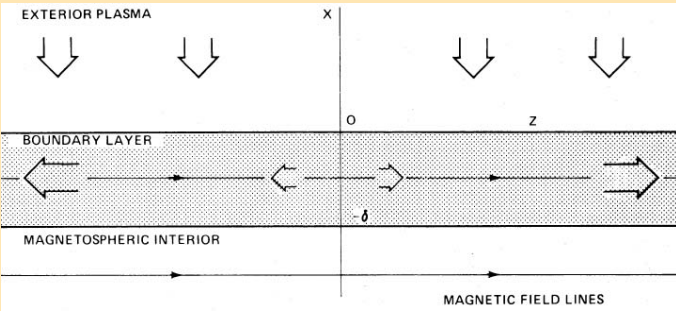
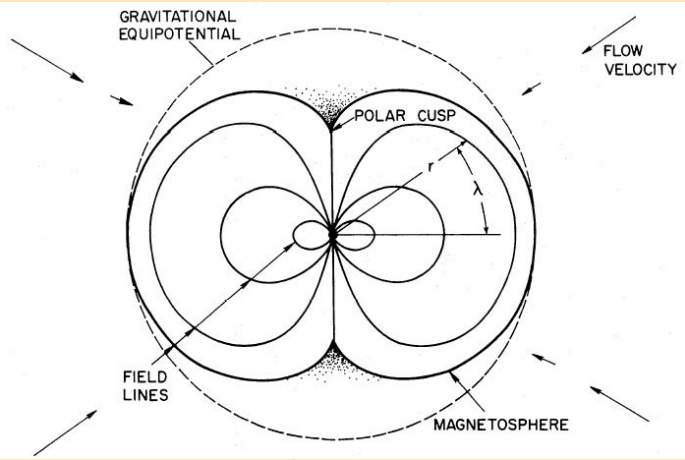
$$|K_{\text{sd}}| = \frac{k_t \mu^2}{(r_m r_{\text{cor}})^{3/2}} \geq 2\pi I |\dot{\nu}_{\text{sd}}^{\text{obs}}|$$

$$r_m \leq \left( \frac{k_t \mu^2}{2\pi I |\dot{\nu}_{\text{sd}}^{\text{obs}}|} \right)^{2/3} \left( \frac{\omega_s^2}{GM_{\text{ns}}} \right)^{1/3}$$

Name	$B_{12}$ , G	$P_s$ , s	$ \dot{\nu}_{\text{sd}} $ , $10^{-13}$ Hz/s	$r_m$ , $10^8$ cm	$r_A$ , $10^8$ cm	$r_m/r_A$
Vela X-1	2.6	283	3	1.5	8.3	0.18
4U 1907+09	2.1	438	0.4	2.9	6	0.48
GX 301-2	4	683	1	1.9	5.5	0.34
X Persei	3.3	837	0.24	4.5	23	0.2

$$r_m \leq (0.2 - 0.5) r_A$$

# Diffusion of the accreting material into the stellar field



$$\dot{m}_{\text{diff}}(r_m) = \begin{cases} \frac{L_X R_{\text{ns}}}{GM_{\text{ns}}} = \dot{m}_a \\ 4\pi r_m \delta_m(r_m) \rho(r_m) v_{\text{ff}}(r_m) \end{cases}$$

$$\delta_m(r_m) = \left[ t_{\text{ff}}(r_m) D_{\text{eff}}(r_m) \right]^{1/2}$$

$$\dot{m}_{\text{diff}} \sim \frac{\mu^2 D_{\text{eff}}^{1/2}}{(GM_{\text{ns}})^{3/4}} \left( \frac{c_s(r_m)}{v_{\text{ff}}(r_m)} \right)^{-2} \left[ r_m \right]^{-15/4}$$

$$D_{\text{eff}}(r_A) \simeq \frac{\dot{m}_{\text{diff}}^2(r_A) (GM_{\text{ns}})^{3/2}}{\mu^4} \left( \frac{c_s(r_A)}{v_{\text{ff}}(r_A)} \right)^4 \times r_A^{15/2} \left[ \frac{r_m}{r_A} \right]^{15/2}$$



“Penetration problem” for  $r_m \equiv r_A$

$$D_{\text{eff}}(r_A) \simeq 10^{17} \text{ cm}^2 \text{ s}^{-1} \times \mu_{30}^{2/7} m^{3/7} \mathfrak{M}_{15}^{-1/7} \left( \frac{c_s(r_A)}{v_{\text{ff}}(r_A)} \right)^4 \left[ \frac{r_m}{r_A} \right]^{15/2}$$

Coulomb diffusion  $D_C = \frac{c^2}{4\pi\sigma} \sim 2 \times 10^{-16} D_{\text{eff}}(r_A)$

Bohm diffusion  $D_B = \frac{c k_B T_i(r_m)}{16 e B(r_m)} \sim 3 \times 10^{-10} D_{\text{eff}}(r_A)$

Is the Magnetospheric radius **smaller** than the **Alfvén** radius ?

$$r_m \simeq 0.054 r_A \times \mu_{30}^{-4/105} m^{-6/105} \mathfrak{M}_{15}^{2/105} \left( \frac{c_s(r_m)}{v_{\text{ff}}(r_m)} \right)^{-8/15} \left( \frac{D_{\text{eff}}(r_m)}{D_B(r_m)} \right)^{2/15}$$

$$r_m \simeq 0.6 r_A \times \mu_{30}^{-4/105} m^{-6/105} \mathfrak{M}_{15}^{2/105} \left( \frac{c_s(r_m)}{0.01 v_{\text{ff}}(r_m)} \right)^{-8/15} \left( \frac{D_{\text{eff}}(r_m)}{D_B(r_m)} \right)^{2/15}$$

# Spin-down rates of LPXPs

$$\left| \dot{\nu}_{\text{sd}}^{\text{max}} \right| \times 10^{-12} \text{ Hz s}^{-1}$$

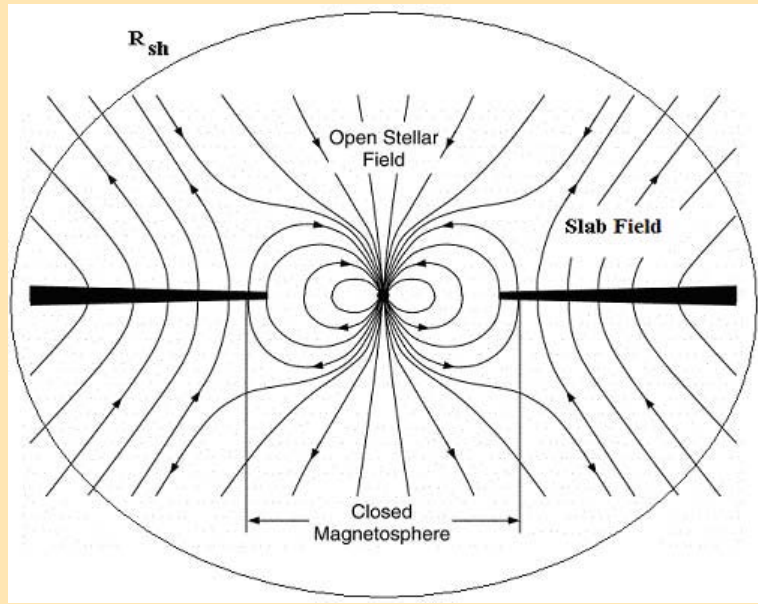
Name	$B_{12}, \text{ G}$	$P_s, \text{ s}$	$\left  \dot{\nu}_{\text{sd}}^{\text{obs}} \right $	$\left  \dot{\nu}_{\text{sd}}^{(0)} \right $	$\left  \dot{\nu}_{\text{sd}}^{(t)} \right $	$\left  \dot{\nu}_{\text{sd}}^{(m)} \right $
OA0 1657-415	3.2	38	3.2	0.056	0.18	3.3
Vela X-1	2.6	283	0.2	0.0003	0.012	0.4
4U 1907+09	2.1	438	0.04	0.0002	0.008	0.2
4U 1538-522	1.8	529	0.06	0.0002	0.008	0.15
GX 301-2	4	683	0.1	0.003	0.02	0.7
X Persei	3.3	837	0.024	0.0001	0.0013	0.03

$$\left| \dot{\nu}^{(0)} \right| = \frac{1}{2\pi I} \left[ \frac{\mu^2}{r_{\text{cor}}^3} \right]; \quad \left| \dot{\nu}^{(t)} \right| = \frac{1}{2\pi I} \left[ \mathfrak{M} \omega_s R_A \right]; \quad \left| \dot{\nu}^{(m)} \right| = \frac{1}{2\pi I} \left[ \frac{\mu^2}{(r_{\text{ma}} r_{\text{cor}})^{3/2}} \right]$$

$$\left| \dot{\nu}_{\text{sd}}^{(0)} \right| < \left| \dot{\nu}_{\text{sd}}^{(t)} \right| \ll \left| \dot{\nu}_{\text{sd}}^{\text{obs}} \right| < \left| \dot{\nu}_{\text{sd}}^{(m)} \right|$$

## Torque exerted on NS

$$2\pi I\dot{\nu} = K_a + K_b + K_c$$



- Inside the magnetosphere:

$$K_a = \dot{M} \ell(r_{\text{ma}}) \simeq \dot{M} \omega_s r_{\text{ma}}^2$$

- At the magnetospheric boundary

$$K_b = \frac{k_t \mu^2}{(r_{\text{ma}} r_{\text{cor}})^{3/2}} \left( \frac{\Omega_f(r_{\text{ma}})}{\omega_s} - 1 \right)$$

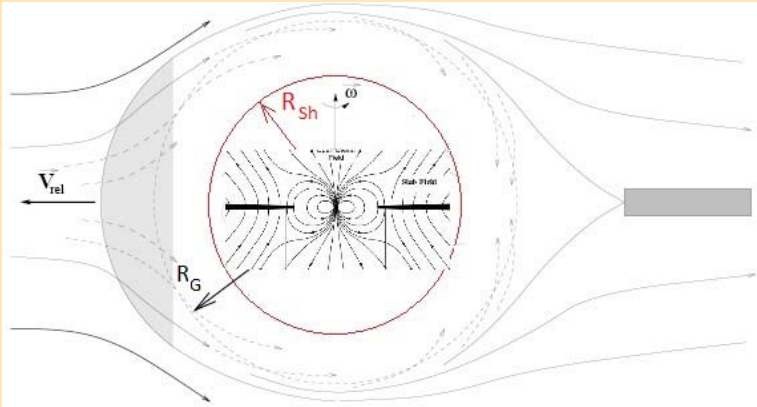
- Beyond the magnetospheric boundary

$$K_c = K_b \times \left( \frac{r_{\text{ma}}}{r} \right)^{3/2} \quad (r > r_{\text{ma}})$$

# EQUILIBRIUM PERIOD

$$P_{\text{eq}} = P_{\text{f}}(r_{\text{ma}}) \left[ 1 - \frac{1}{\sqrt{2}k_{\text{t}}} \left( \frac{r_{\text{ma}}}{r_{\text{A}}} \right)^{7/2} \right] \simeq 0.5 A_{\text{m}} P_{\text{orb}}$$

$$A_{\text{m}} = F(\xi, \beta_0, c_{\text{so}}, \mu, \dot{\mathfrak{M}}, v_{\text{rel}}, M_{\text{ns}})$$



Angular velocity of matter at  $r_{\text{ma}}$

$$\Omega_{\text{f}}(r_{\text{ma}}) = \xi \Omega_{\text{orb}} \left( \frac{r_{\text{G}}}{R_{\text{sh}}} \right)^2$$

$$2\pi I \dot{\nu} = \left[ \dot{\mathfrak{M}} \omega_{\text{s}} r_{\text{ma}}^2 \right] + \left[ \frac{k_{\text{t}} \mu^2}{(r_{\text{ma}} r_{\text{cor}})^{3/2}} \left( \frac{\Omega_{\text{f}}(r_{\text{ma}})}{\omega_{\text{s}}} - 1 \right) \right]$$

# Minimum / Maximum possible equilibrium period

- Minimum possible equilibrium period (weakly magnetized wind  $\beta_0 \sim \beta_{\max}$ )

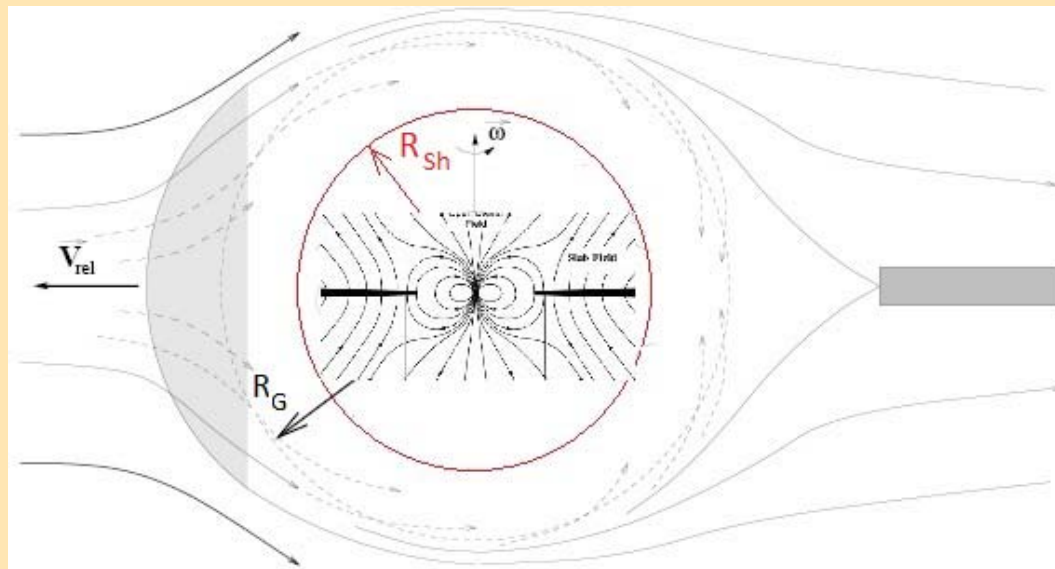
$$P_{\min}^{\text{eq}} \simeq 14 \text{ s} \times \mu_{30}^{6/7} \mathfrak{M}_{15}^{-3/7} m^{-5/7}$$

- Maximum possible equilibrium period (strongly magnetized wind  $\beta_0 \sim 1$ )

$$P_{\max}^{\text{eq}} \simeq 20 \text{ s} \times P_{\text{orb(d)}}^{15/7} \xi_{0.2}^{-15/7} \beta_0^{-12/7} c_6^{24/7} m^{-8/7}$$

$$P_{\min}^{\text{eq}}$$

$$R_{\text{sh}} \sim r_{\text{A}}$$

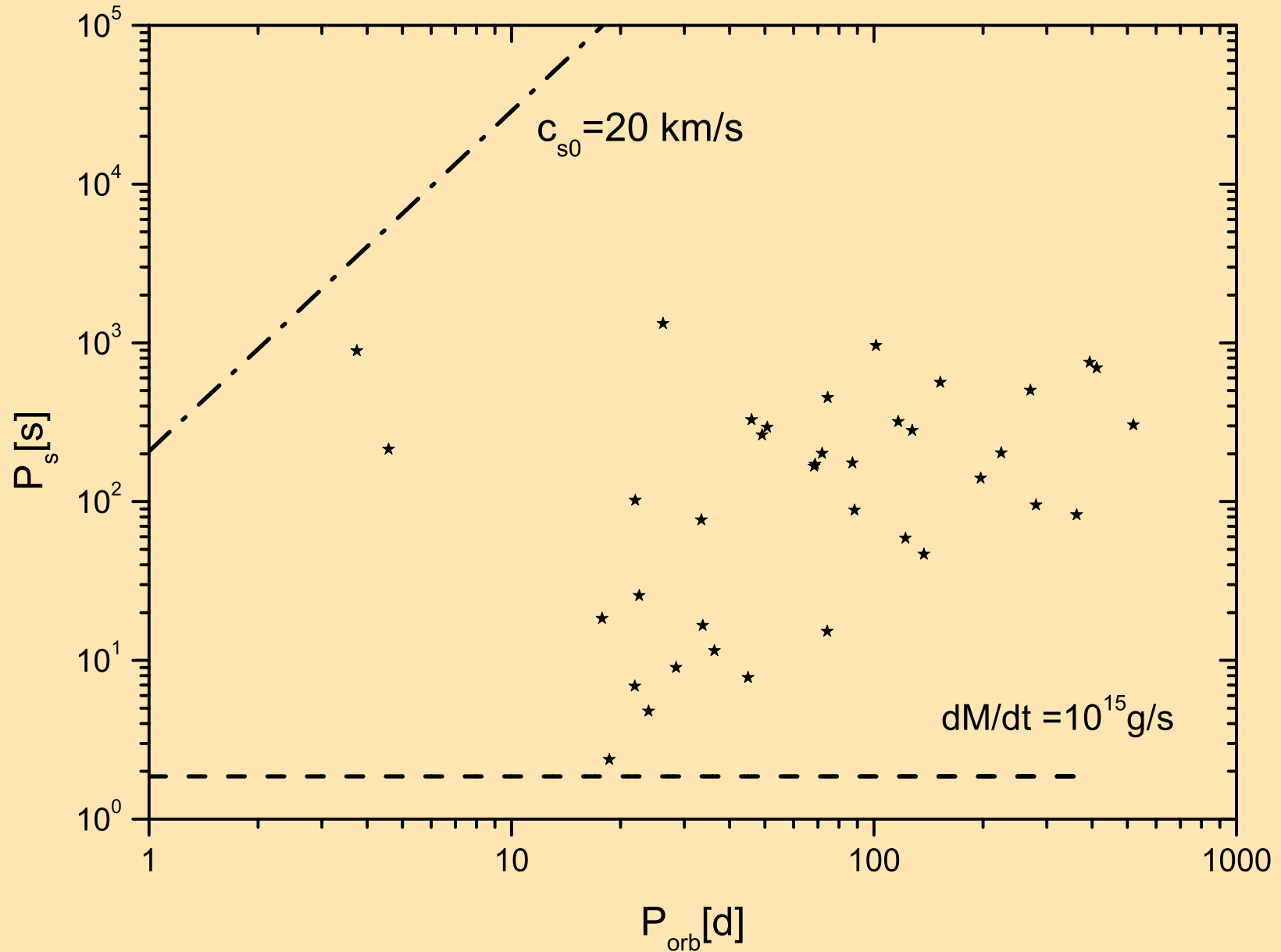


$$P_{\max}^{\text{eq}}$$

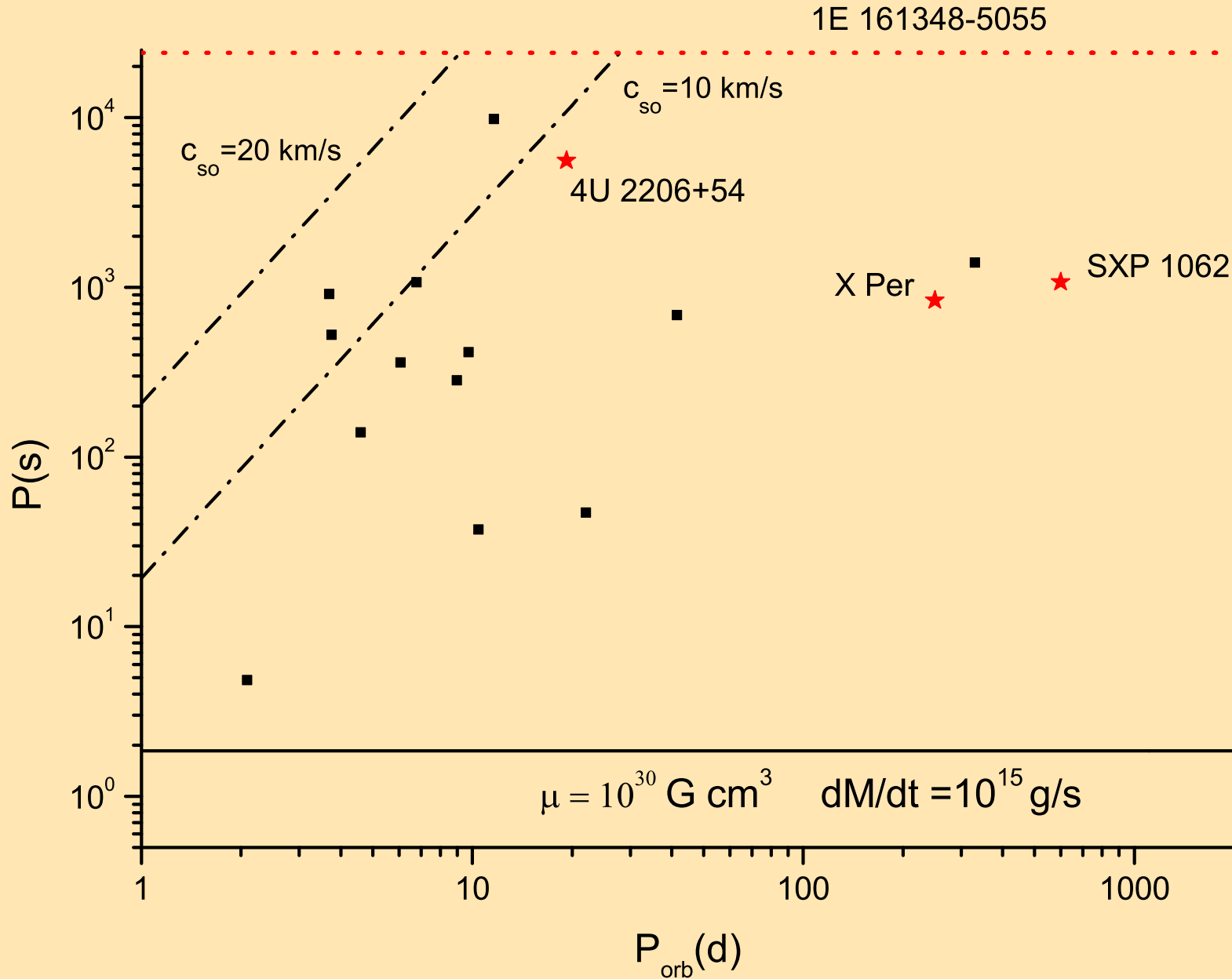
$$R_{\text{sh}} \gg r_{\text{A}}$$

# $[P_s \text{ vs. } P_{\text{orb}}]$ diagram of Be/X-ray pulsars in SMC

$$B \sim 10^{11} - 10^{12} \text{ G}, \quad \dot{M} \sim 10^{14} - 10^{15} \text{ g/s}$$



# Equilibrium periods range in ML-accretion scenario



## Magnetic-Levitation Accretion onto a Neutron Star

1. **Accretion** from a **magnetized** wind ( $1 \leq \beta_0 \leq \beta_{\max}$ )
2. **Deceleration** of a free-falling material at the **Shvartsman radius**  $R_{\text{sh}}$
3. **Formation** of a **non-Keplerian Magnetically-Levitating Disk (MAGLEV Disk)**
4. **Diffusion** of **accreting material** into the **stellar MF** at the magnetospheric boundary

<b>Maglev radius</b>	$r_{\text{mlv}} = \beta_0^{-2/3} \left( \frac{c_s(r_G)}{v_{\text{rel}}} \right)^{4/3} r_G$
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New parameters:

<b>Magnetospheric radius</b>	$r_{\text{ma}} = \left( \frac{c m_p^2}{16 \sqrt{2} e k_B} \right)^{2/13} \frac{\alpha_B^{2/13} \mu^{6/13} (GM_{\text{ns}})^{5/13}}{T_0^{2/13} L_x^{4/13} R_{\text{ns}}^{4/13}}$
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<b>ML-torque</b>	$K_{\text{ml}} = k_m \frac{\mu^2}{(r_{\text{ma}} r_{\text{cor}})^{3/2}} \left( \frac{\Omega_{\text{ml}}(r_{\text{ma}})}{\omega_s} - 1 \right)$
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## Popular simplifications of MLA scenario:

### Quasi-spherical (QS) and Keplerian disk (KD) accretion scenarios

1. Neglecting Magnetic field in Stellar Wind of Massive Star we come to

- QS scenario if  $r_{\text{circ}} < r_A$ , ( $v_{\text{rel}} > v_{\text{cr}}$ ), or
- KD scenario if  $r_{\text{circ}} > r_A$ , ( $v_{\text{rel}} < v_{\text{cr}}$ )

This implies the surface magnetic field of O/B star to be weak ( $B_* \ll 100$  G) and  $R_{\text{sh}} = 0$

2. Magnetospheric radius  $r_{\text{ma}} \rightarrow r_A$  for the case

$$\left\{ \begin{array}{l} \frac{\mu^2}{2\pi r_{\text{ma}}^6} = \rho(r_{\text{ma}}) c_s^2(r_{\text{ma}}) \\ \dot{m}_{\text{in}}(r_{\text{ma}}) = \frac{L_X R_{\text{ns}}}{GM_{\text{ns}}} \end{array} \right. \quad \left| \quad \begin{array}{l} \rho(r_{\text{ma}}) \rightarrow \rho_{\text{sp}} = \frac{\dot{m}}{4\pi r^2 v_{\text{ff}}} \\ c_s(r_{\text{m}}) \rightarrow v_{\text{ff}}(r_{\text{m}}) \end{array} \right. \quad \left\{ \begin{array}{l} \frac{\mu^2}{2\pi r_A^6} = \rho_{\text{sp}}(r_A) v_{\text{ff}}^2(r_A) \\ r_A = \left( \frac{\mu^2}{\dot{m} (2GM_{\text{ns}})^{1/2}} \right)^{2/7} \end{array} \right.$$

3. The price is paid: difficulties with explanation of

- X-ray luminosity and rapid spin evolution of accretion-powered pulsars,
- origin of turbulence and large-scale magnetic field in a Keplerian accretion disk,
- a contribution of non-thermal processes into spectra of HMXBs.

