Twisted photons and electrons as a new tool for the investigations

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in collaboration with

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V.P. Kosheleva, M.E. Groshev

What can be twisted?

- Photons
  Б.А. Князев и В.Г. Сербо, УФН 188, 508 (2018) *(review)*
- Electrons
  K.Y. Bliokh *et al.*, Phys. Rep. 690, 1 (2017) *(review)*

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  K.Y. Bliokh *et al.*, Phys. Rep. 690, 1 (2017) *(review)*
- Atoms
  V.E. Lembessis *et al.*, PRA 89, 053616 (2014) *(Th.)*
- Neutrons
  C.W. Clark *et al.*, Nature 525, 504 (2015) *(Exp.)*
- Gravitational Waves
  I. Bialynicki-Birula, Szymon Charzynski, PRL 121, 171101 (2018) *(Th.)*

Outline

- Twisted photons
  - Definition
  - Experimental realization
  - Applications

- Twisted electrons
  - Definition
  - Theoretical description
  - Applications

- Conclusions

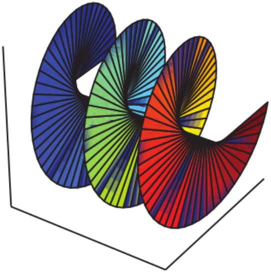
Terminology

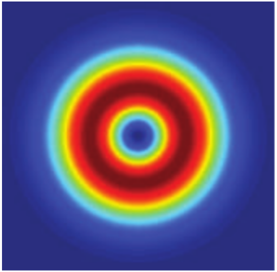
Twist; Vortex; Phase dislocation; Phase singularity

What are twisted photons?

Short answer

Light beams that carry orbital angular momentum (OAM) $l\hbar$





Features

- Poynting vector spirals along the direction of propagation
- Phase in the transverse plane of $e^{il\varphi}$

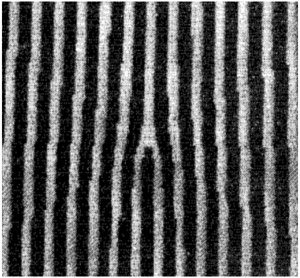
Experimental realization

Forked diffraction gratings

V.Y. Bazhenov *et al.*,
JETP Lett. 52, 429 (1990)

Computer-generated holograms

N.R. Heckenberg *et al.*,
Opt. Lett. 17, 221 (1992)



Detailed study

L. Allen *et al.*, PRA 45, 8185 (1992)

Optical range

Review

A.M. Yao and M.J. Padgett, Adv. Opt. Photon. 3, 161 (2011)

Range of OAM ($l\hbar$)?

Optical range

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Range of OAM ($l\hbar$)

$5000\hbar$	Y. Shen *et al.*, J. Opt. 15, 044005 (2013)
$10000\hbar$	R. Fickler *et al.*, Proc. Natl. Acad. Sci. 113, 13642 (2016)

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No limits!

XUV range and x-ray

Compton back-scattering

U.D. Jentschura and V.G. Serbo, PRL 106, 013001 (2011) *(Th.)*

High-harmonic generation

F. Kong *et al.*, Nat. Commun. 8, 14970 (2017) *(Exp.)*
D. Gauthier *et al.*, Nat. Commun. 8, 14971 (2017) *(Exp.)*

Free electron laser

E. Hemsing *et al.*, PRL 106, 164803 (2011) *(Th.)*
E. Hemsing *et al.*, Nature Phys. 9, 549 (2013) *(Exp.)*

Helical undulator

M. Katoh *et al.*, PRL 118, 094801 (2017) *(Th.)*
T. Kaneyasu *et al.*, PRA 95, 023413 (2017) *(Exp.)*

Applications

Multi-dimensional entanglement

R. Fickler *et al.*, Science 338, 640 (2012)

Quantum communication

G. Vallone *et al.*, PRL 113, 060503 (2014)

Biophysics

D.G. Grier, Nature 424, 810 (2003)

Traps

M. Liu *et al.*, Nature Nanotechnol. 5, 570 (2010)

Twisting of light around rotating black holes

F. Tamburini *et al.*, Nature Phys. 7, 195 (2011)

Applications on atomic level

Scattering

S. Stock *et al.*, PRA 92, 013401 (2015) *(Th.)*
L. Zhang *et al.*, PRL 117, 113904 (2016) *(Th.)*

Absorption (Excitation and Ionization)

A. Afanasev *et al.*, PRA 88, 033841 (2013) *(Th.)*
H.M. Scholz-Marggraf *et al.*, PRA 90, 013425 (2014) *(Th.)*
A. Surzhykov *et al.*, PRA 91, 013403 (2015) *(Th.)*
C.T. Schmiegelow *et al.*, Nat. Commun. 7, 12998 (2016) *(Exp.)*
R.A. Müller *et al.*, PRA 94, 041402 (2016) *(Th.)*
D. Seipt *et al.*, PRA 94, 053420 (2016) *(Th.)*
A. Surzhykov *et al.*, PRA 94, 033420 (2016) *(Th.)*
T. Kaneyasu *et al.*, PRA 95, 023413 (2017) *(Exp.)*

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“Transfer of optical orbital angular momentum to a bound electron”

Details

Single trapped ion $^{40}\text{Ca}^+$
Laser near 729 nm
Holographic plates (OAM $= \pm\hbar$)

$$\begin{aligned} \left|4^2 S_{1/2}, m_J = -1/2\right\rangle &\rightarrow \left|3^2 D_{5/2}, m_J = 3/2\right\rangle \\ \left|4^2 S_{1/2}, m_J = -1/2\right\rangle &\rightarrow \left|3^2 D_{5/2}, m_J = -5/2\right\rangle \end{aligned}$$

C.T. Schmiegelow *et al.*, Nat. Commun. 7, 12998 (2016)

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C.T. Schmiegelow *et al.*, Nat. Commun. 7, 12998 (2016)

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New selection rules!

Atom absorbed two quanta of angular momentum from a single photon!

C.T. Schmiegelow *et al.*, Nat. Commun. 7, 12998 (2016)

What are twisted electrons?

Short answer

Solutions of the free Dirac equation in cylindrical coordinates

What are twisted electrons? Details

Short answer

Solutions of the free Dirac equation in cylindrical coordinates

Quantum numbers

E	Energy
p_z	**p** projection onto the z axis
j_z	**j** (TAM) projection onto the z axis
μ	Helicity

What are twisted electrons? Details

Explicit form

$$\psi_{\varkappa m p_z \mu}(\mathbf{r}) = \int \frac{e^{im\varphi_p}}{2\pi p_\perp} \delta(p_\parallel - p_z)\delta(p_\perp - \varkappa) i^{\mu - m} \psi_{\mathbf{p}\mu}(\mathbf{r}) d\mathbf{p}$$

Notations

$\varkappa$	$\sqrt{\varepsilon^2 - 1 - p_z^2}$
m	$\mathbf{j}$ projection onto the z axis
μ	Helicity
$\psi_{\mathbf{p}\mu}$	Plane wave

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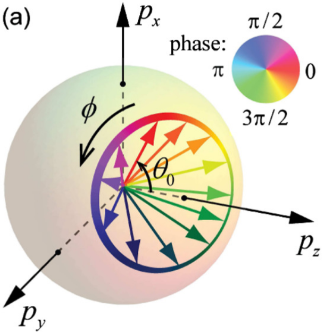

What are twisted electrons?

Predicted

K.Y. Bliokh *et al.*, PRL 99, 190404 (2007)

Realized

J. Verbeeck *et al.*, Nature 467, 301 (2010)
M. Uchida and A. Tonomura, Nature 464, 737 (2010)
B. J. McMorran *et al.*, Science 331, 192 (2011)

Motivation

- additional degree of freedom m
- magnetic moment $= m\mu_B$
- spin-orbit interaction increases (m up to 1000)[a]

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[a] E. Mafakheri *et al.*, Appl. Phys. Lett. 110, 093113 (2017)

Applications

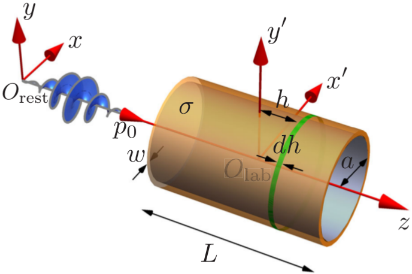
- Landau Levels and Aharonov-Bohm States
  K.Y. Bliokh *et al.*, PRX 2, 041011 (2012) *(Th.)*
- Larmor and Gouy rotations
  G. Guzzinati *et al.*, PRL 110, 093601 (2013) *(Exp.)*
- Transition radiation from a magnetic moment
  I.P. Ivanov and D.V. Karlovets, PRL 110, 264801 (2013) *(Th.)*
- Magnetic monopole field
  A. Beche *et al.*, Nature Phys. 10, 26 (2014) *(Exp.)*
- Measure electron magnetic circular dichroism
  J. Rusz *et al.*, PRL 113, 145501 (2014) *(Th.)*

Recent review

K. Y. Bliokh et al., Phys. Rep. 690, 1 (2017)

Measurement of Orbital Angular Momentum

- H. Larocque *et al.*, PRL 117, 154801 (2016) *(Th.)*



- T.R. Harvey *et al.*, PRA 95, 021801 (2017) *(Th.)*
  "Stern-Gerlach-like approach to electron orbital angular momentum measurement"

Vavilov-Cherenkov radiation

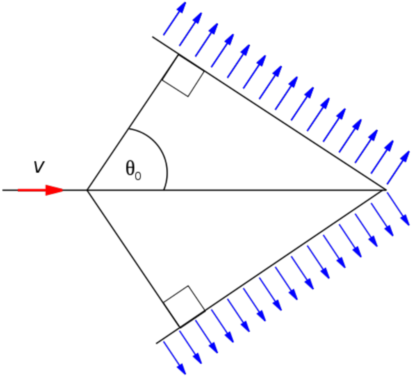
Reminder

Vavilov-Cherenkov radiation emitted by an electron passing through a medium at a speed greater than the speed of light

$$\cos\theta_0 = \frac{1}{vn} + \frac{\omega}{2E}\frac{n^2-1}{vn}$$

Notations

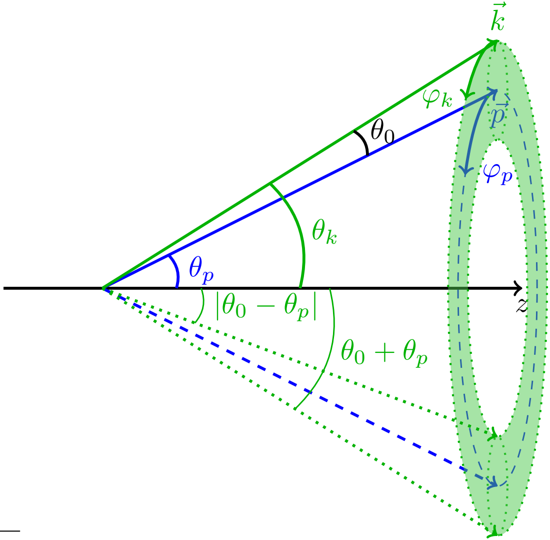
E	Energy of the electron
v	Velocity of the electron
n	Refraction index
ω	Energy of the photon



Vavilov-Cherenkov radiation by twisted electrons

Parameters

E	300 keV
v	0.78
λ_{ph}	550 nm (green)
ω	2.25 eV
n	1.33 (water)
θ_0	14.5°

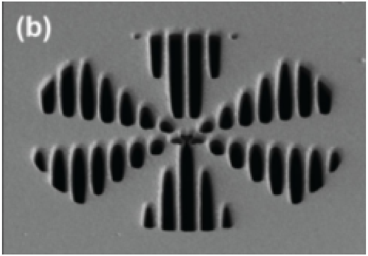


I.P. Ivanov *et al.*, PRA 93, 053825 (2016)

Superposition of two twisted states

Wave function

$$\psi_{\varkappa p_z E \mu}(\mathbf{r}) = \frac{1}{\sqrt{2}}\left[\psi_{\varkappa m_1 p_z E \mu}(\mathbf{r}) + \psi_{\varkappa m_2 p_z E \mu}(\mathbf{r})\right]$$



Spectral-angular distribution

$$\frac{d\Gamma}{d\omega d\Omega}$$

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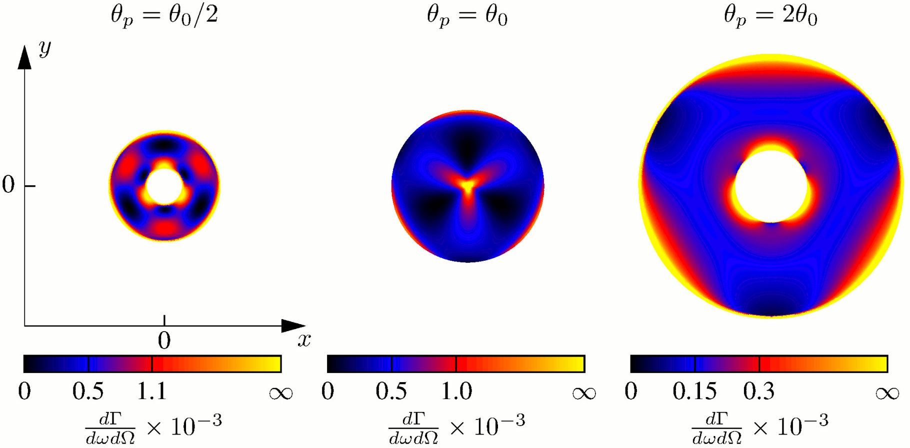

Interaction with ionic and atomic targets

Nonrelativistic

R. V. Boxem *et al.*, PRA 2014,
"Rutherford scattering of electron vortices"

O. Matula *et al.*, NJP 2014,
"Radiative capture of twisted electrons by bare ions"

R. V. Boxem *et al.*, PRA 2015,
"Inelastic electron-vortex-beam scattering"

Relativistic

V. G. Serbo *et al.*, PRA 2015,
"Scattering of twisted relativistic electrons by atoms"

VAZ *et al.*, PRA 2017,
"Radiative recombination of twisted electrons with bare nuclei: Going beyond the Born approximation"

V. P. Kosheleva *et al.*, PRA 2018,
"Elastic scattering of twisted electrons by an atomic target: Going beyond the Born approximation"

Going beyond the Born approximation

Wave function construction

as a solution of the Dirac equation *in the nucleus field* with asymptotics

$$\Psi^{(+)}_{\varkappa m p_z \mu}(\mathbf{r}) \xrightarrow[r\to\infty]{} \psi_{\varkappa m p_z \mu}(\mathbf{r}) + G^{(\mathrm{tw})}\frac{e^{ipr}}{r}$$

the explicit form

$$\Psi^{(+)}_{\varkappa m p_z \mu}(\mathbf{r}) = \int \frac{e^{im\varphi_p}}{2\pi p_\perp}\delta(p_\parallel - p_z)\delta(p_\perp - \varkappa)\, i^{\mu-m}\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r})\, d\mathbf{p}$$

where

$$\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}) \xrightarrow[r\to\infty]{} \psi_{\mathbf{p}\mu}(\mathbf{r}) + G^{(\mathrm{pw})}\frac{e^{ipr}}{r}$$

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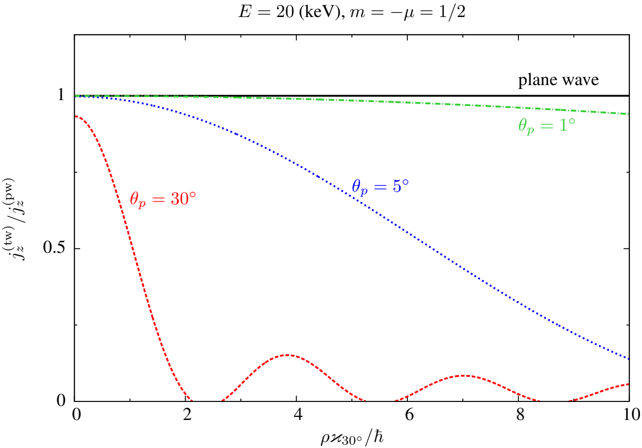
where

$$\Psi^{(+)}_{\mathbf{p}\mu}(\mathbf{r}) \xrightarrow[r\to\infty]{} \psi_{\mathbf{p}\mu}(\mathbf{r}) + G^{(\mathrm{pw})} \frac{e^{ipr}}{r}$$

Problems

Flux

is no more a homogeneous function!



Solutions

integrated probability and flux

$$\frac{d\overline{W}^{(\mathrm{tw})}}{d\Omega_k} = \int f(\mathbf{b}) \frac{dW^{(\mathrm{tw})}}{d\Omega_k}(\mathbf{b}) d\mathbf{b}, \qquad J_z = \int f(\mathbf{r}_\perp) j_z^{(\mathrm{tw})}(\mathbf{r}_\perp) d\mathbf{r}_\perp$$

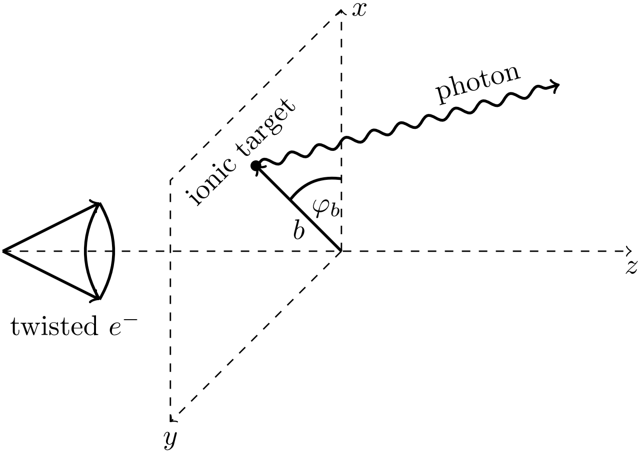
Macroscopic target

Ions distribution is given by $f = N_{\mathrm{ions}}/(\pi R^2)$, then the cross section is

$$\frac{d\sigma^{(\mathrm{tw})}}{d\Omega_k} = \frac{1}{\cos\theta_p} \int_0^{2\pi} \frac{d\varphi_p}{2\pi} \frac{d\sigma^{(\mathrm{PW})}}{d\Omega_k}$$

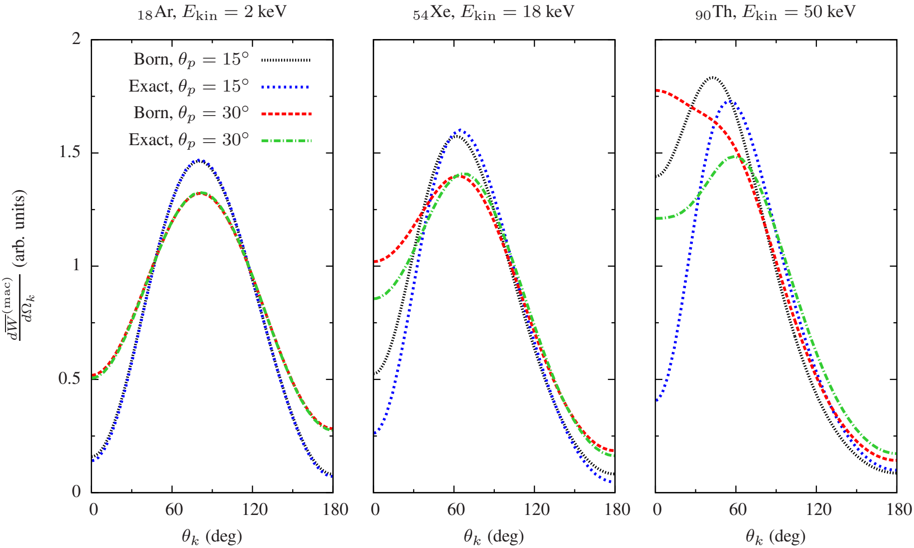
Radiative capture of twisted electrons

$$e^-_{\mathrm{tw}} + A^{Z+} \to A^{(Z-1)+}(1s) + \gamma_{\mathrm{ph}}$$



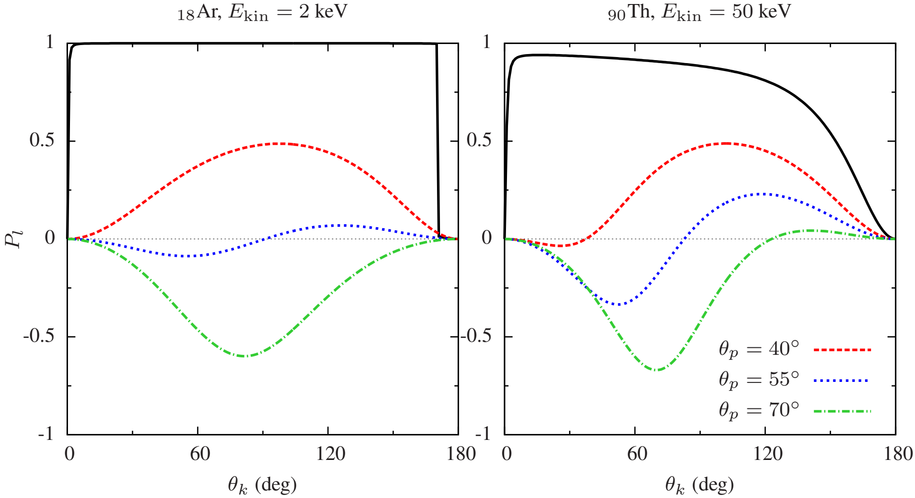
VAZ *et al.*, PRA 95, 012702 (2017)

Angular distribution



VAZ *et al.*, PRA 95, 012702 (2017)

Polarization



VAZ *et al.*, PRA 95, 012702 (2017)

Are the emitted photons twisted?

- Average value of the projection of the TAM operator

$$\langle \mathbf{J} \cdot \hat{\mathbf{n}}_0 \rangle = \frac{Tr\left[\rho^{(\mathrm{ph})} \rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0} (\mathbf{J} \cdot \hat{\mathbf{n}}_0)\right]}{Tr\left[\rho^{(\mathrm{ph})} \rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}\right]}$$

$\rho^{(\mathrm{ph})}$	density operator of the photon
$\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}$	describes the detector

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VAZ *et al.*, PRA 97, 043808 (2018)

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- Opening angle

$$\cos\theta_\gamma = \frac{1}{\omega} \langle \mathbf{p} \cdot \hat{\mathbf{n}}_0 \rangle ,$$

VAZ *et al.*, PRA 97, 043808 (2018)

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$$\cos\theta_\gamma = \frac{1}{\omega}\langle \mathbf{p}\cdot\hat{\mathbf{n}}_0\rangle\,,$$

- Dispersions Δ_J and Δ_p

$$\Delta_J = \sqrt{\left\langle\left(\mathbf{J}\cdot\hat{\mathbf{n}}_0\right)^2\right\rangle - \langle \mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle^2}$$

VAZ *et al.*, PRA 97, 043808 (2018)

Radiative electron capture

Simplest case: $\hat{\mathbf{n}}_0 = \hat{\mathbf{e}}_z$

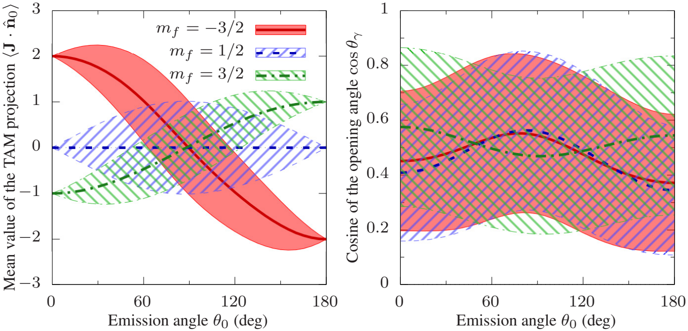
$$\langle J_z \rangle = \mu - m_f \,, \quad \Delta_J = 0$$

VAZ *et al.*, PRA 97, 043808 (2018)

Radiative electron capture

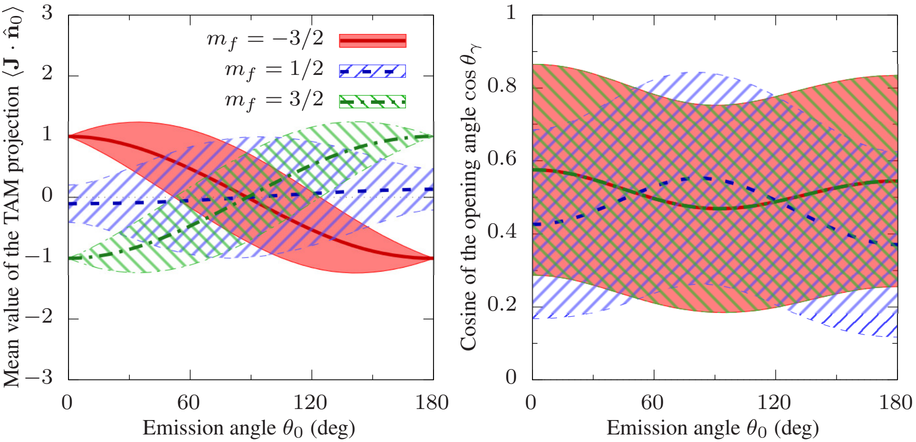
$\hat{\mathbf{n}}_0 \neq \hat{\mathbf{e}}_z$

Recombination of the 2 keV plane-wave electron with $\mu = 1/2$ into the $2p_{3/2}(m_f)$ state of the H-like Ar ($Z = 18$)



VAZ *et al.*, PRA 97, 043808 (2018)

Radiative capture of *unpolarized* electron



VAZ *et al.*, PRA 97, 043808 (2018)

Conclusions

Main feature

Nonzero projection of the OAM

- Additional degree of freedom
- Enhancement mechanisms
- Bigger amount of information
- New selection rules

As a result

More powerful investigation tool

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Thank you!