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Variations of the solar radius from transits of Mercury through the Sun's disk

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М. Л. Свешников. Вариации радиуса Солнца из прохождений Меркурия по диску Солнца.

Ключевые слова: прохождения Меркурия, радиус Солнца, активность Солнца.

Представлены результаты анализа моментов внутренних контактов прохождений Меркурия по диску Солнца 1631–1973 гг. Обнаружено присутствие 80- и 11-летних циклов в вариациях радиуса. Амплитуда 80-летнего цикла составляет $0.24'' \pm 0.05''$. Амплитуда 11-летнего цикла равна $0.08'' \pm 0.02''$, причем существует положительная корреляция между вариациями радиуса Солнца и числом солнечных пятен. Указано на противоречие с результатами Laclare [19] для знака корреляции. Найденное вековое уменьшение радиуса Солнца, равное $-0.06'' \pm 0.03''$, связывается с систематическими ошибками наблюдений. Обсуждены возможные источники ошибок.

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Keywords: Mercury, solar radius, solar activity.

The results of analysis of internal contacts timings for transits 1631–1973 are presented. The presence of a 80- and 11-year cycle in radius' variations are detected. The amplitude of 80-cycle is found to be about $0.24'' \pm 0.05''$, and the amplitude of 11-cycle equals $0.08'' \pm 0.02''$. There is a positive correlation between 11-year variations of the solar radius and sunspot numbers. The contradiction with Laclare's results [19] in the sign of correlation is discussed. Obtained secular decrease of the solar radius, equal $-0.06'' \pm 0.03''/cy$, is attributed to systematic errors of observations. Possible sources of errors are discussed.

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Contents

1. Introduction

Investigation of global oscillations of the Sun's figure is an additional source of information on its inner structure. Up to the 20th century study of the solar figure was restricted to estimations of its apparent oblateness. However in the beginning of the 20th century this problem started to be treated as connected with the problem of the solar activity, which is characterized by Wolf numbers. The detection of the solar neutrino deficiency in Brookhaven ^{37}Cl-experiment in the 1970's, elaboration of the gravitational theories, alternated to GRT, and appearance of helioseismology have activated researches on the solar astrometry.

Classical meridian and micrometric measurements of the solar diameter, observations of solar eclipses and Mercury's transits through the Sun's disk are available from the middle of the 17th century, heliometric measurements from the middle of the 18th century and, finally, astrolabe observations from the 1970's. There exist also measurements based on the analysis of isophote distribution on the solar disk. It has to be emphasized that the solar astrometry is an extremely complicated problem. The main difficulties are the big apparent size of the Sun, its significant surface brightness, limb-darkening (i.e. uncertainty of the solar edge position), specific temperature regime during observations and significant influence of the atmosphere. For example the effective refraction index might be changing almost twofold within a few minutes [1]- [2].

Meridian observations (especially those of the 17th-18th centuries) are disturbed by all the above factors. Moreover such measurement can be carried out once a day at the zenith distance changing systematically during the year. That is why they have poor accuracy and noticable systematic errors. Micrometric measurements can be done several times a day, but they are under influence of the same factors and provide approximately the same accuracy. Astrolabe observations (tangency of the direct and mirrored images) provide the better accuracy than the meridian ones. These observations are made with more stable instrument, twice in a day on the same zenith distance, that leads to more realistic account of irradiation and atmospheric effects. Observations of the solar eclipses are not so sensitive to the atmospheric effects, but they are effected by the errors of the lunar marginal zone. In addition, very complicated theory of the orbital motion and rotation of the Moon is to be used. Processing of the contact observations for Mercury's transits are more simple from this point of view, but these phenomena are much more rare.

Analysis of long time series of the solar and geophysical data allowed to detect a large number of different periods in solar activity in the range from several minutes to decades and even hundreds of years [3]– [4]. As far as in 1813 Wurm has discovered by the processing of 6 solar eclipses that the solar radius $R_\odot$ is oscillating around its ephemeris value from $-5''$ to $+3''$. However, only in the 1920's the problem of variations of $R_\odot$ started to be considered in the close connection with the solar activity variations. Naturally, the solar 11-year cycle and secular variations are of a special interest.

In the classic paper [5] Eddy and Boornazian have obtained the significant $\dot{R}_\odot = -1.12''/cy$ from the Greenwich meridian measurements. However this result was disproved by other works, dealing with analysis of eclipses and transits (table 1).

Table 1. Determination of the secular decrease of the solar radius.

Authors	Period	Method	$\dot{R}_\odot [''/cy]$
Eddy, Boornazian[1] [5]	1836−1953	meridian	−1.12
Parkinson et al.[2] [6]	1715−1973	trans.+eclip.	-0.14 ± 0.08
Gilliland[3] [7]	1715−1973	trans.+eclip.	−0.1
	1836−1975	+meridian	
Dunham et al.[4] [8]	1715−1979	eclipses	-0.13 ± 0.07
Dunham et al.[4] [8]	1925−1980	eclipses	$\Delta R_\odot = -0.7$
Sofia et al.[5] [9]	1836−1937	meridian	$\mid \dot{R} \mid < 0.25$
Shapiro [10]	1736−1973	transits	$\mid \dot{R} \mid < 0.15$
		+meridian	(0.02 ± 0.07)
Krasinsky et al. [11]	1715−1973	transits	-0.25 ± 0.08
Ribs et al. [12]	XVII−XVIII	meridian	−2
Morrison et al. [13]	1715−1980	eclipses	$\mid \dot{R} \mid < 0.1$

Notes:
1 The decrease of the solar vertical diameter is found as $-0.36''/cy$.
2 Periodical components in variation of $R_\odot$ are detected.
3 20- and 76–80-year variations of radius are detected.
4 Decrease of the solar radius by $0.7''$ in the last 50 years.
5 The same observations with [5].

General revision of the meridian, micrometric, heliometric and astrolabe measurements of the solar diameter was made by Toulmonde in 1997 [14].

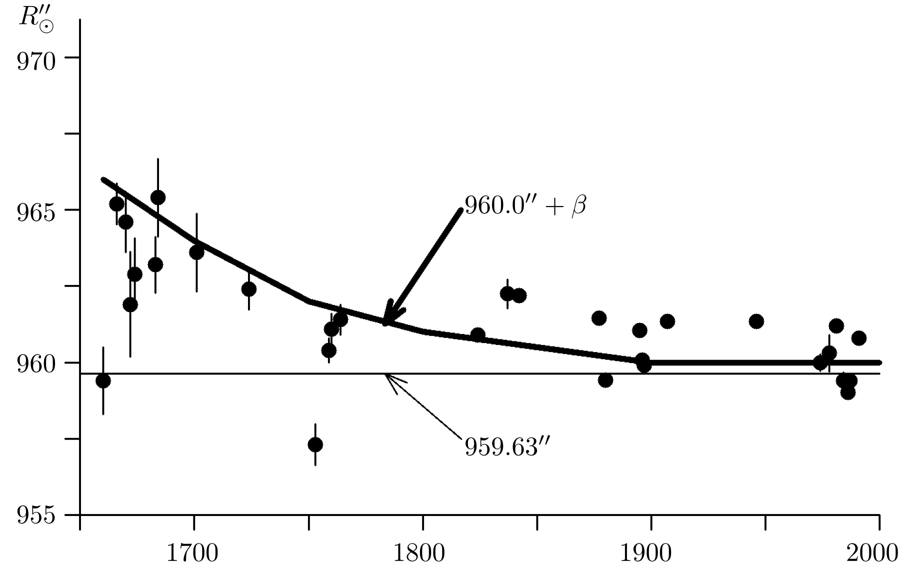

Fig. 1. Influence of the instrumental corrections on the solar radius value: $\beta = 15''/D$ before 1870; $\beta = 0.7''/D$ after 1870; D[cm] is diameter of telescope (from [14]).

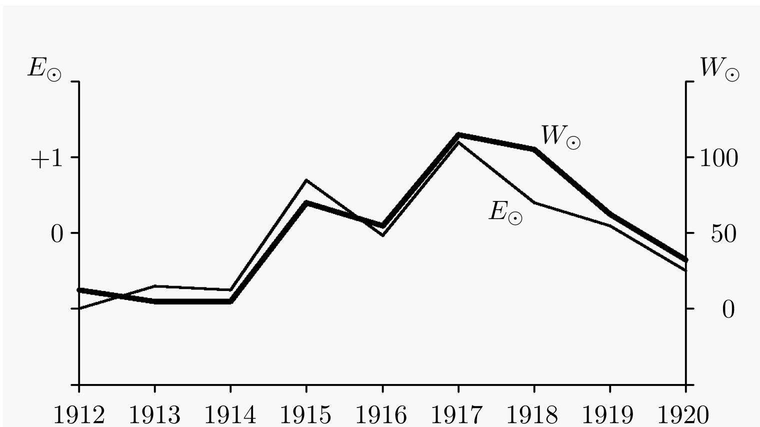

Fig. 2. Comparison of Wolf numbers $W_\odot$ and the normalized variations of the solar constant $E_\odot$ from Mount Wilson observations in 1912–1920 [15].

Analysis of 30 series of observations included more 60,000 measurements for 1600-2000 and showed that the marked secular decrease of the solar radius is caused mainly by poor account of diffraction, the value of which is varying with changing of diameter of instruments used during 300-year observational interval (fig. 1). Averaging of the original (non-reduced) data gives $R''_{\odot} = (960.83 \pm 0.23) - (0.70 \pm 0.30)(T - 19.00)$, closed to the result of [5].
After the reduction, the processing of the homogeneous set of observations does not show any noticeable $\dot{R}_{\odot}$ and gives $R^m_{\odot} = 960.0'' \pm 0.1''$ as average value for the solar semi-diameter on the whole time interval.

In the beginning of the 20th century a positive correlation between variations of the solar constant $E_{\odot}$ and Wolf numbers $W_{\odot}$ has been found (fig. 2) [15].

Results of the investigation of the connection between solar radius variations and the solar activity are more contradictory. Though the existence of the 11-year visible variations of the solar radius is unquestionable for both modern and historical observations [12], [16], the sign of its correlation with $W_{\odot}$, shown in the last column in Table 2, varies from author to author.

Table 2. Determination of 11-year oscillations of the solar radius.

Authors	Period	Method	$\Delta R_{\odot}('')$	Corr.
Meyermann, 1950 [17]		meridian	0.09	+
Rubashov et al., 1972 [18]	1836–1937	meridian		+, −?
Gilliland, 1981[1] [7]	1715–1973	mer.+ecl.	0.1	−
Delache et al., 1985 [20]	1975–1984	astrolabe		−
Wittmann et al.,1993[2] [21]	1981,1991	meridian	0.4	+
Ulrich et al.,1995 [22]	1982–1994	at 545 nm		+
Laclare et al., 1996[3] [19]	1975–1994	astrolabe	0.11	−
Noel, 1997 [23]	1990–1995	astrolabe		+
Basu, 1998[4] [24]	1660–1990	mer.+ast.		+
Rozelot, 1998[5] [25]	15 years	various	0.3 − 0.7	+

Notes:.
[1] From the Mercury transits the 80-year cycle with the amplitude $\Delta R_{\odot} = 0.2'' \pm 0.1''$ has been discovered also.
[2] Both measurements are made in the vicinity of the maximums of activity.
[3] In several papers Laclare detected a few periods with negative correlation $\Delta R_{\odot}$ with respect to $W_{\odot}$.
[4] Data from [14], [23].
[5] Review of papers from the last 15 years.

From high dense set of astrolabe observations in 1974–1995 (obs. Calern, SERGA) Laclare et al. [19] has found

$$R_\odot = 959.40'' + 0.04'' \sin 2\pi t/T + 0.10'' \cos 2\pi t/T, \tag{1}$$

($t = JD - 2446120.5$ и $T = 10.5^a$) with the negative correlation between $\Delta R_\odot$ and $W_\odot$. It is in accordance with Gilliland's result [7]. Analogous result is obtained by Delache et al. [20] from a comparison of measurements of the solar radius with several solar activity indices.

On the other hand, the observations made in 1990–1995 by Noel [23] (Santiago, CCD-astrolabe) show clear positive correlation $\Delta R_\odot$ and $W_\odot$ for the 11-cycle. Comparison of the historical observations of the solar semi-diameter with $W_\odot$ by Basu [24], based on homogeneous set by Toulmonde [14], leads to the same result (fig. 3).

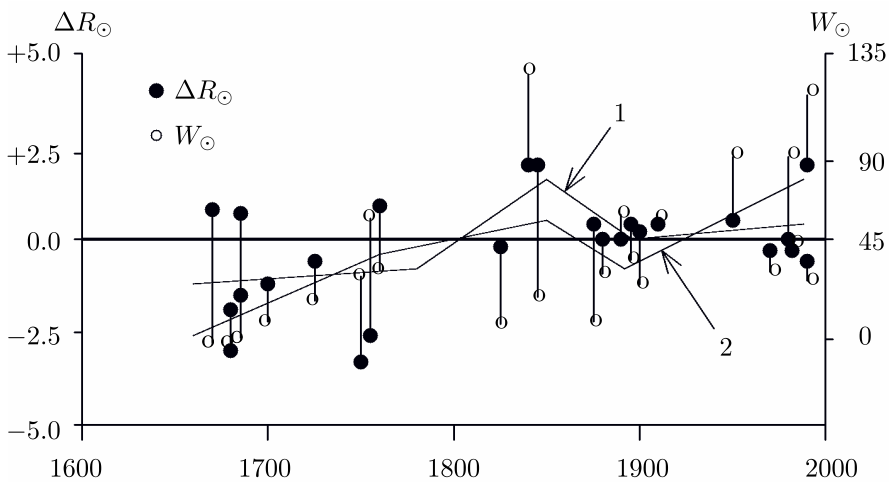

Fig. 3. Correlation of the solar radius variations around $R_\odot = 960.0''$ and Wolf numbers around $W_\odot = 45.9$ by Basu (correlation equals $+0.37-0.40$; the averaged curves: $1 - \Delta R_\odot$; $2 - W_\odot$).

Roselot [25] has analyzed the results of measurement in the last 15 years and drew to conclusion that the amplitude of the variation of $R_\odot$ is less $0.7''$ and in the last 5 years it is most probably about $0.3''$ (with error $0.1''$). Perhaps there exists a contribution to the oblateness of the Sun, changing with the solar activity cycle. Furthermore the solar profile (helioid) is unstable and has a complicated pulsing form, i.e. the global geometry of the Sun depends on its radiation. In particular Roselot gives the following empirical relation between the solar radiation, Wolf number and the solar radius value, expressed in angular seconds:

$$E_\odot(Wm^{-2}) = 539.14 \ + \ 0.0084W_\odot \ + \ 0.87R''_\odot. \qquad (2)$$

Roselot makes the following remark about observations of Laclare's group: "main conclusion is the remarkable likeness between the irradiance and diameter time series when a time lag of about half a solar cycle period is applied".

The paper by Gilliland [7] gave birth to a common opinion, that Mercury's transits are not applicable or are poor effective to search the 11-cycle solar radius variations since the mean data separation is 9 years, so being to large.

The present paper deals with investigation of the mentioned above components of the solar radius variations by an independent analysis of Mercury's transits on 300-year interval.

2. Reduction of observations

As the first stage an archive of observations of Mercury's transits through the solar disk has been collected in IAA RAS [26]- [29]. The archive includes about 4500 moments of contacts for 39 phenomena in the time interval 1631–1973, 2700 of them being the internal contacts. In the archive there are moments of ingress and egress of the Mercury's center on the solar disk. Descriptions of observations were taken from original publications, and only in the case of inaccessible sources they were transcribed from other compilations: LeVerrier [30], Newcomb [31], Morrison and Ward [32]. Information about every observation contains observer's geographical coordinates (λ, φ, h), topocentric moment of contact in original time scale (t_o), name of the observer, telescope parameters, description of a method and conditions of the observation.

Published timing were reduced to the geocenter and UT in the following way:

$$UT_{red} = t_o + \Delta_1 + \Delta_2(\lambda) + \Delta_3(\lambda, \varphi, h), \qquad (3)$$

where Δ_1 is a reduction of t_o for instrumental effects, $\Delta_2(\lambda)$ is a conversion to UT-scale, and $\Delta_3(\lambda, \varphi, h)$ is a reduction to the geocenter.

Some peculiarities of the reduction are given below.

Usually, the moments of contacts in the 17th–18th centuries were published in the local apparent solar time; in the 19th century – in the local mean solar time; in the 20th century – in UT scale (i.e. in Greenwich mean solar time reckoned from midnight). Sometimes other types of time scales were used. When the conversion is carried out from the original time scale to UT, an error in the longitude of place enters directly to the reduced moment. Therefore the error in the longitude should not be greater than a few time seconds. In most cases the observer coordinates for old observations were taken from "Connaissance des Temps pour 1913" or "The Nautical Almanac and Astronomical Ephemeris for year 1913", but sometimes they were determined from geographical atlases. For altitude a change of 1° would change the time of contact by less than 1^s (accuracy of the altitude determinations in 18th century was better than $2'$).

In the 17th century the calibration of the laboratory clock was often made by altitude observation of the Sun or stars. As the refraction and parallax corrections were necessary, for reduction of such observations positions of objects were taken from modern ephemerides. Influence of uncertainty of the adopted value $\Delta T^s = ET - UT$ is small, because the rate of change of the declination of the Sun is only $0.7''/min$ (for right ascension it is about $10''/min$) in the beginning of May or November, and the declination is used only as argument of the trigonometric functions. Totally, the error of determination of the moment in this method does not exceed $15^s - 20^s$.

For reduction of the moment to the geocenter the rigorous formulas for parallax were applied. It should be mentioned that the differential formula was often used for Δ_3 in Lagrange's form [33]:

$$\Delta_3(\lambda, \varphi, h) = T_{geo} - T_{topo} = -A\rho' \sin\varphi' - B\rho' \cos\varphi' \cos(C + \lambda), \qquad (4)$$

where A, B, C are functions of parallaxes of the Sun, Mercury and the sidereal time. It may lead to significant errors due to simplification of expressions for A, B, C. So the formula (4) is valid only for the case of near central transits with small value of the minimal distance between the Sun's and Mercury's disk centers ρ_{min}. For almost tangential transits the error in the reduction could be as much as several hundreds of seconds. At the same time a half of the observed transits has fraction $\rho_{min}/R_\odot$ more 0.5.

The main bulk of visual observations of the internal contacts is so-called "black drop" effect, i.e. the dark bar between the limbs of the Sun and Mercury. This phenomenon shifts the observed moment of internal contact with respect to the geometrical one. Near contacts during about one minute the continuous set of the phases are observed, which are each other transformed. However, if some clearly marked phases ("planet is already (still) on the disk" and/or "the geometrical contact with the black drop" and etc.) are excluded, the rest phases fill a relatively short interval in 15^s-20^s. Unfortunately their sequence for the second and the third contacts will be different. For example in 1878 the sequence of phases in Paul's notation [34] for the second contact was marked as $HIKAEMCB$, but for the third contact it became $HAMIBCKE$. This points out the random nature of the phase sequence. In this case the best way to determine the mean moment is a simple averaging over all the moments.

The drop effect can be caused by different physical effects: the solar limb darkening, image blurring of the Sun and Mercury by the atmosphere and diffraction, and the contrast phenomenon, i.e. reaction of the eye's retina on variations of illumination on its surface. The process of the appearance of black drop was discussed in numerous papers. The most important contributions are, in our opinion, by Lalande (1774), H.Struve (1874), von Kuhl (1929), Wittmann (1974). We carried out also the analytical and numerical simulation of formation of Mercury's image near the solar limb [36].

It may be thought that no reduction for the drop phenomenon is needed, since systematic delay in timing of the 2sd contact will be cancelled in the analysis by an equal error in timing of the 3rd contact [35]. But it is not the case. Firstly the amount of observations of the 3rd contact is by about 50% greater that of the 2nd contact. Secondly the observational conditions of 2nd and 3rd contacts are different. As a result the black drop phenomenon may leed to a fictitious diminuation of the observed semi-diameter of the Sun.

Evidently the correction to the observed moment distorted by the drop can be expressed by the simple formula:

$$\Delta T_d^s \simeq \frac{d''}{v\sqrt{1-(\rho_{min}/R_\odot)^2}}, \tag{5}$$

where d'' is the ephemeris distance between limbs in arcsec at the moment of visible contact with the drop (it depends on observation conditions); v''/s is the velocity of Mercury measured along a chord of its apparent path. For May and November transits $v = 1/10''/s$ and $1/15''/s$ correspondingly. From numerical simulation it was obtained that for the statistical averaged observational conditions for wich the blurring spot radius is equal $1.5'' - 2''$ the value d'' is about $1.4''$.

In 1868 Stone proposed for the drop reduction the following formula: $\Delta T_d^s = (r''/30)/\dot{\rho}$, where r'' is the apparent radius of Mercury, and $\dot{\rho}$ is the velocity of approaching of limbs. Wolf and Andre [37] carried out a series of experiments for determination of errors in moments of contacts and have found:

- errors rapidly grow with decreasing of telescope diameter;
- influence of magnification is smaller than that of seeing;
- telescopes with a good objective and $D > 20$ cm produce practically no errors in moments of contact;
- size of the planet disk has neglecting influence upon the error value.

For telescopes at Paris observatory the limbs separation was $\simeq 1''$ when apparent contact was observed. Taking into account the velocity of limbs approaching for transit in 1868 we obtain $\Delta T_d^s = 16^s$. From the Stone's relation one obtains 2.6^s for the same conditions ; numerical simulation gives $16^s - 20^s$.

Drop phenomenon distorts the Gaussian distribution $O-C$ for the moments of contacts and can generate the bimodal curve of the distribution (for example, as for 2nd contacts in 1914 and 1957 and for 3rd contacts in 1907 and 1953). Unfortunately observers, especially in the 20th century, often do not accompany the observed moments by descriptions of the corresponding phases. This complicates interpretation of these observations. Nevertheless in some cases the reduction has led to decrease of scattering of O-C in comparison with the simple averaging of moments. For instance in the case for 1878 transit, when the standard error of $O-C$ after reduction became twice smaller comparing with the classical result by Paul [34]. More detailed description of the drop phenomenon and some recommendations for the observers are given in [36] (in particularly it is proposed to record two phases: the apparent contact and break of the drop: their half-sum is approximately the moment of the geometrical contact).

Observations of the first contact are useless due to the factor of “abruptness” of appearance of Mercury on the solar disk. Moments of the 4th contact may be used in some cases (sometimes Newcomb indeed used them [31]). Under certain conditions the accuracy of observation of the 4th contacts may be of the same order as that of internal contacts. Paul [34] proposed a procedure of estimation of the mean moment of 4th contact based on analysis of the set of observations, taking advantage of high quality of the equipment used. However our experience has shown that the Paul’s procedure is not very reliable. Thus, the observations of the fourth and partially the first contacts were used only for testing of quality of observation [26], [27].

Summarizing, one can say that the errors in calculations of mean moments of contacts are in the interval $15^s - 30^s$ for the 17th century and $1^s - 2^s$ for the 20th century. Due to the slow apparent velocity of the Mercury relative to the Sun ($\simeq 0.07 - 0.1''/s$) the accuracy of determination of the distance between limbs is about $\simeq 0.1''$ for the 19th century and $\simeq 0.5'' - 0.7''$ for the 17th century. It is one order better than the accuracy of meridian measurements of Mercury and the Sun. That is why Newcomb [31] affirmed a danger rather to underestimate the transits in comparison with meridian observations than overestimate them.

3. Conditional equations for transits

For processing of the observational data the package ERA [38] was used. The ephemerides DE405 [39] were used as a standard theory. Only 2nd and 3rd contacts were processed. Generally the algorithm of computing of ephemeris moment of contacts is adapted from Bessel's theory of solar eclipses, in which the planet is substituted for the Moon ([33], [31], [40], etc.). In our paper [41] and in [35] the residuals $O-C$ are calculated directly from the apparent topocentric coordinates of the Sun and Mercury and their apparent size. It makes possible to take into account some effects more correctly than in [31]. Analysis $O-C$ provided information for estimation of quality and relative weight of each observation within the set of observations of the contact. In table 3 normal points of the geocentric contacts are given. The columns in table 3 are: the year and month of the event, integer (JD_o) and fractional (jd_2, jd_3) parts of the Julian date for the averaged observed geocentric moment of contact in UT-scale, $(O-C)^s$ with the standard error for DE405 and the number of observations (N) participated in averaging. Plots for the averaged $O-C$ are shown on fig. 4–5.

From the analysis of $(O-C)$ one can detect unreliable normal points of internal contacts.

1631 (egress of the center). It is the first observation of Mercury on the solar disk made by P.Gassendi. The observation was carried out with the screen with the solar image diameter about 20 cm. Moment of contact is determined by measurement of altitude of the Sun. Probable error of $O-C=-93^s$ is $\pm 20^s$.

1661 (ingress of the center). The observation was made by J.Hevelius. The moment of ingress of the center is obtained by recalculation of six measurements of distances between centers of the Mercury and the Sun made along the apparent path on the projection to the screen. The position of Mercury that is the closest to the limb was observed to be about 11% of its total path length on the solar disk. The original records were used to determine moment of observations from altitudes of the Sun and stars. Probable error of $O-C=-55^s$ is $\pm(20^s-30^s)$.

1677. The observations of all contacts were made by Ed.Halley with a telescope of poor quality and by Townley at a place with rather uncertain coordinates. Gallet's observation is evidently wrong.

1697. Newcomb could not found the original record of the Cassini's observation, so it was taken from LeVerier's work [30]. These observations were made on the screen and had poor accuracy, although the moment of the 3rd contact is close to its ephemeris value.

1707. The observations are rejected completely. Due to mistaken prediction of region and the time of visibility of the event by de Lahir the single trustable observation of an egress of the Mercury from the solar disk was made by Roemer, but his estimation of the moment of the egress of the center was very rough: $O - C = -100^s$.

1736 (observations of the center). All measurements were made under very poor conditions.

1740. The single observation of this transit, made by Wintrop, was nearly tangential and was completely rejected: the observation had considered by Wintrop as very doubtful.

1756. The transit was observed in Peking by inexperienced observers with poor equipment. A few obtained moments of contacts contain large errors. This is clearly seen from the results of the observations of the second contact $O - C = +46^s$.

1782. The transit is a unique one, because the relation of distance between the centers to the value of the solar radius is about 0.97. The interval between external and internal contacts is longer than 7^m, so phases of the event are enormously expanded in time and are very difficult to register. Average moments of the both contacts have large errors. It is interesting, that for the 2nd contact $(O - C)_{II} = -9^s$, i.e. it is less than average value of $O - C$ for transits of that period time. On the other hand for the 3rd contact $(O - C)_{III} = -33^s$, and it is greater than average value of $O - C$ for the neighbour transits by the same value of order $\simeq 10^s$. It could be explained by decreasing of the solar radius about $0.''2$. Value of $(O-C)_{IV}$ is very large and equal to -90^s.

1786. The observations of the second contact are scanty and contradictory, which may be explained by the low altitude of the Sun (about $3^\circ - 7^\circ$). For observation of the egress of Mercury a large number of small telescopes was used.

1937. This is a tangential transit (relation $\rho_{min}/R_\odot > 1$), i.e. only the first and the fourth contacts can be observed. There are information on the 4th contact, observed at Cape observatory with $O - C = -128^s$.

1957. The transit is close to a tangential one. There are a few unreliable observation of the second contact.

Although a large number of papers deal with processing of observations of transits, the coefficients of conditional equations are given only in [30], [31], [35], [41]. In paper [43] several mistakes in such expressions has been done. The derivation of the equations is based on the relations for coordinates of the center of the Earth relative to the umbral and penumbral cones from the Mercury and the heliocentric ecliptic coordinates of the Earth and Mercury. Equations include corrections to orbital elements of Mercury and the Earth, corrections to the adopted values of Mercury's and the

Sun's semi-diameters, determining the size of the umbral and penumbral on Bessel's plane, and unknowns dependent on due to long-term variation of the Earth's rotation. In this paper the conditional equations for the moments of contacts are chosen in the form proposed by Morrison and Ward [35]:

$$\sin(Q-i)(V+\dot{V}T)+\sin(Q+i)(W+\dot{W}T)\pm\cos(Q\mp i)(N+\dot{N}T)+$$

$$+(\tfrac{1}{r}-\tfrac{1}{r_o})\Delta R_\odot+0.911\tfrac{\rho}{r}\dot{D}T^2k=\tfrac{\rho}{r}(\Delta\sigma+\dot{D}\Delta T^s). \qquad (6)$$

where

$$\Delta R_\odot=\Delta R^o_\odot+\dot{R}_\odot T+\sin(2\pi\varphi_{80})\Delta R^s_{80}+\cos(2\pi\varphi_{80})\Delta R^c_{80}$$

$$+\tfrac{W_{\odot,m}}{100}[\sin(2\pi\varphi_{11})\Delta R^s_{11}+\cos(2\pi\varphi_{11})\Delta R^c_{11}]. \qquad (7)$$

Because of the poor accuracy of observed positional angle of contacts the corresponding equation for the angle of contact [41] is not considered.

The right hand side of equation (6) contains:

— $\dot{D}$ is a apparent rate of approaching of the centers of Mercury and Sun at the moment of contact, expressed in $[''/s]$. It differs from the velocity of motion of the Mercury along the apparent trajectory by the factor $\sqrt{(1-(\rho_{min}/R_\odot)^2}$. The value of $\dot{D}$ can be calculated also by numerically from the formulas, including the topocentric apparent rights ascensions, declinations and geocentric distance of the Sun ($\alpha_\odot$, $\delta_\odot$) and the Mercury (α, δ):

$$\begin{aligned}
\cos D &= \sin\delta_\odot\sin\delta + \cos\delta_\odot\cos\delta\cos(\alpha-\alpha_\odot),\\
\cos P\sin D &= \cos\delta_\odot\sin\delta - \sin\delta_\odot\cos\delta\cos(\alpha-\alpha_\odot),\\
\sin P\sin D &= \cos\delta_\odot\sin(\alpha-\alpha_\odot).
\end{aligned}$$

Here P is the positional angle of the contact.

— $\Delta\sigma$ is an ephemeris angular geocentric distance between the limbs of Mercury and the Sun at the observed moment of the contact and expressed in seconds of arc. It is calculated from the difference of observed and the ephemeris time of the contact as $\Delta\sigma=-(O-C)^s\cdot\dot{D}$. Otherwise $\Delta\sigma$ can be obtained as $\Delta\sigma=D-(R_\odot-R)$ where $R_\odot$ and R are the apparent semi-diameters computed from adopted values at unit distance: $R_\odot=959.63''/\rho_\odot$, and $R_\odot=3.36''/\rho$, where $\rho_\odot$ and ρ are geocentric distances of the Sun and Mercury correspondingly.

— $\Delta T^s=ET-UT$ is a correction to UT scale in seconds, consistent with the adopted value of the tidal deceleration of the Moon $\dot{n}_m=-26''/cy$. Values ΔT^s were obtained in [56] and are published in "The Astronomical Almanac" (p.K8–K9).

— r is a heliocentric distances of the Mercury.

Table 3. The mean observed geocentic moments of the internal contacts.

Year month	Date JD_o	Contact II (UT) jd_2	$(O–C)^s$	N	Contact III (UT) jd_3	$(O–C)^s$	N
1677.11	2333882	0.898416	$-5.6^\star$	1	1.116587	$-28.2^\star$	1
1697.11	2341183				0.821426	$-15.9^\star$	1
1723.11	2350685	0.101967	-22.5 ± 1.2	8			
1736.11	2355435	0.882222	-32.5 ± 3.5	9	0.992229	-20.3 ± 3.7	6
1743.11	2357985	0.843278	-25.9 ± 2.4	5	1.031311	-17.7 ± 3.6	8
1753.05	2361455				0.920814	-19.1 ± 2.9	18
1756.11	2362736				0.786577	-65.5 ± 4.9	2
1769.11	2367487	0.307474	-17.9 ± 2.3	7	0.506871	-11.9 ± 7.1	4
1782.11	2372238	0.112930	$-\ \ 8.4 \pm 6.0$	20	0.159393	-33.4 ± 3.6	16
1786.05	2373506	0.625211	-27.8 ± 13.7	3	0.848220	-18.1 ± 2.6	32
1789.11	2374788	0.036889	-14.6 ± 2.6	22	0.239053	-16.4 ± 2.5	6
1799.05	2378257	0.881802	-19.2 ± 1.7	59	1.187830	-19.2 ± 1.4	46
1802.11	2379538				0.986928	-11.9 ± 1.3	35
1822.11	2386839	0.544144	-28.6 ± 6.3	4	0.656438	-16.6 ± 4.7	7
1832.05	2390308	0.877426	-18.2 ± 2.0	25	1.157415	-10.1 ± 1.8	34
1845.05	2395060	0.183214	-20.3 ± 2.2	29	0.450739	-17.6 ± 2.3	17
1848.11	2396340	0.963055	-15.3 ± 1.7	33	0.186242	$-\ \ 9.1 \pm 1.8$	2
1861.11	2401091	0.722428	-24.2 ± 2.6	4	0.887727	-11.6 ± 1.6	37
1868.11	2403641	0.727726	-13.3 ± 2.8	4	0.875114	$-\ \ 4.1 \pm 1.1$	79
1878.05	2407111	0.135999	$-\ \ 4.4 \pm 1.0$	202	0.447026	$+\ \ 2.9 \pm 1.2$	102
1881.11	2408392	0.429638	$+\ \ 4.1 \pm 1.3$	16	0.649959	$+\ \ 2.2 \pm 1.9$	14
1891.05	2441862	0.499341	$-\ \ 3.5 \pm 2.7$	24	0.697380	$+11.2 \pm 2.5$	79
1894.11	2413143	0.165348	$-\ \ 2.7 \pm 1.9$	54	0.382879	$+\ \ 1.0 \pm 1.5$	54
1907.11	2417893	0.934801	-19.9 ± 1.7	53	1.074457	$-\ \ 5.0 \pm 1.3$	94
1914.11	2420443	0.916013	-24.4 ± 1.6	52	1.088241	-16.6 ± 1.2	59
1924.05	2423913	0.407224	-34.8 ± 2.9	21	0.732840	-17.2 ± 1.5	94
1927.11	2425194	0.627534	-26.3 ± 2.1	23	0.852410	-25.2 ± 1.6	91
1940.11	2429945	0.368553	-20.4 ± 0.9	123	0.577528	-25.3 ± 1.2	65
1953.11	2434696	0.153244	-27.9 ± 1.9	88	0.254722	-38.2 ± 2.3	70
1957.05	2435964	0.506250	-17.8 ± 5.2	15	0.596852	-42.1 ± 6.6	19
1960.11	2437246	0.108261	-30.2 ± 2.0	71	0.298568	-37.5 ± 1.4	77
1970.05	2440715	0.682169	-42.4 ± 4.2	27	1.006760	-45.5 ± 2.0	149
1973.11	2441996	0.825648	-51.1 ± 3.9	22	1.052404	-46.6 ± 1.4	150

* — probably error of timing is about $\pm(15 \div 20)^s$.

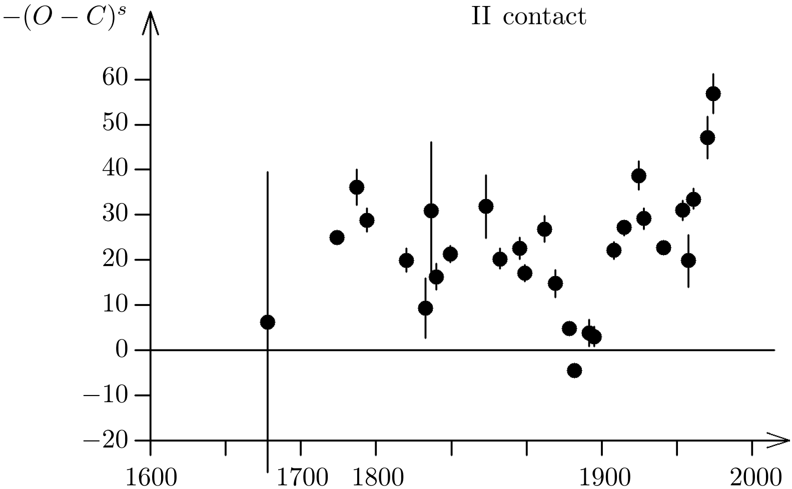

Fig. 4. The normal points of $(O-C)^s$ for second contacts.

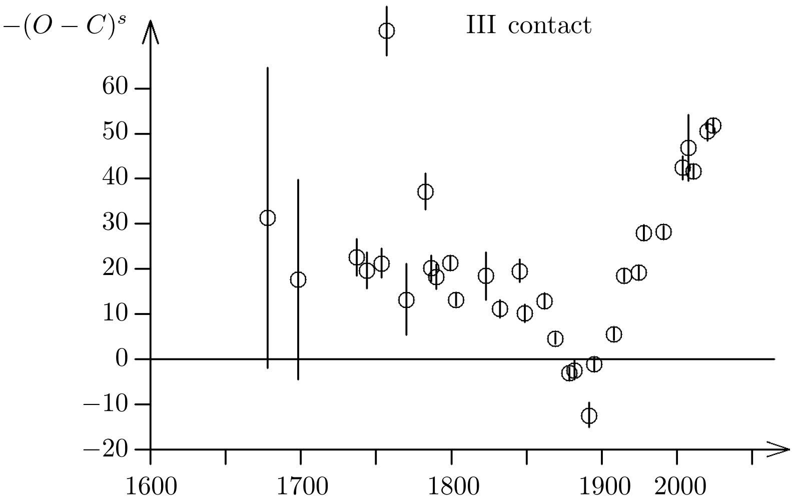

Fig. 5. The normal points of $(O-C)^s$ for third contacts.

Let us consider the left part of equation (6).

The essential feature of the geometry of the transits is that the mean anomalies of the Earth and Mercury in May and November change in the narrow range about 5°-width. Therefore only linear combinations of the corrections to orbital elements, which are functions of mean anomalies, can be determined. These linear combinations are called Newcomb's parameters V, W, N and they are different for May and November transits [31]. The unknown V is the average correction in longitude for all the November transits and W for the May ones. N is principally the correction to the longitude of the node of Mercury's orbit at both types of transits. The upper sign refers to the November events and the lower one to the May transits. Morrison and Ward [35] noted that Newcomb [31, p.448, eq.(3′)] has omitted the sign minus in front of his coefficient for N in the May transits. The unknowns V, W, N and their secular variations are functions of corrections to Kepler's orbital elements of Mercury and the Earth. Numerical values of coefficients of these linear functions are obtained by averaging of mean anomalies over all sets of observed transits. They were given by Newcomb [31], Clemence [42] and were adjusted slightly in [41]:

$$\begin{aligned}
V &= +1.490\Delta L - 2.381 e\Delta\pi - 1.070\Delta e + 1.218 e_o\Delta\pi_o + 1.615\Delta e_o, \\
W &= +0.717\Delta L + 1.376 e\Delta\pi + 0.913\Delta e - 1.140 e_o\Delta\pi_o - 1.610\Delta e_o, \\
N &= \sin i\ (\Delta\Omega - \Delta l_o), \qquad (8) \\
\Delta l_o &= -2e_o\Delta\pi_o\cos(L_o - \pi_o) + 2\Delta e_o\sin(L_o - \pi_o),
\end{aligned}$$

where ΔL, $\Delta\pi$, $\Delta\Omega$, Δe are the corrections to Kepler's orbital elements for Mercury: mean longitude, longitude of perihelion, longitude of node, and eccentricity; ΔL_o, $\Delta\pi_o$, Δe_o are analogous corrections for the orbital elements of the Earth, and Δl_o is the correction to true longitude of the Earth. The correction to the inclination of Mercury orbit is not included into the linear combinations, since the coefficient of Δi is proportional to $\sin u$ (u is a latitude of Mercury reckoned from the node) and it is less 0.01. One should keep in mind, however, that the Newcomb's equations [31] with unknowns V, W, N were derived for the angular heliocentric distance between centers of the Earth and Mercury Δc. So, for utilization of Newcomb's parameters in their standard interpretation in (6) it is necessary to reduce $\Delta\sigma$ to its heliocentric value. For this purpose the factor ρ/r is introduced.

$\dot{V}$, $\dot{W}$, $\dot{N}$ are secular variations of V, W, N with T measured from the epoch 1900.0 in Julian centuries.

Auxiliary angles used to calculation of V, W, N have the following meaning (fig. 6):

i is inclination of Mercury's orbit to the ecliptic.

P is positional angle of contact, reckoned from the north pole of the Earth's equator P_n;

η is angle between directions to P_n and the north pole of the ecliptic P_e with $\sin\eta = \cos\alpha_\odot \sin\varepsilon$, $(\eta \in [-90°, +90°])$, where $\alpha_\odot$ is right ascension of the Sun, ε is the obliquity of the ecliptic.

ω is angle of the contact in accordance with Newcomb, reckoned from the axis, which is oriented against motion of Mercury and is parallel to the ecliptic plane.

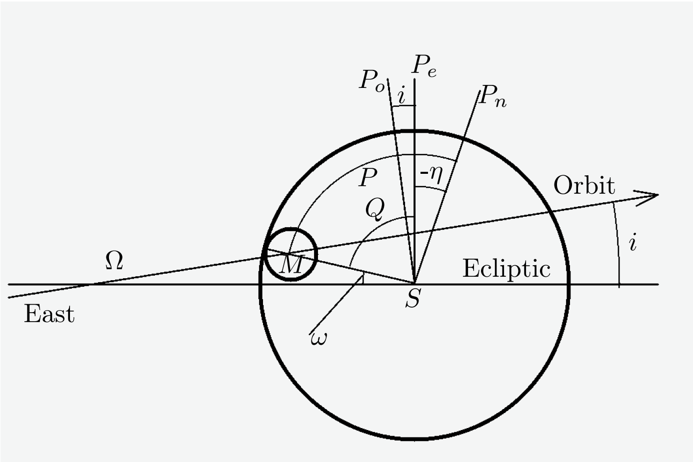

Fig. 6. Position of Mercury (M) on the solar disk for a November transit at the time of second contact [35].

$Q = P + \eta = (270° - \omega)$ is positional angle of the contact, measured eastward from P_e.

A special group of unknowns $\Delta R^o_\odot$, $\dot{R}^o_\odot$, ΔR^s_{11}, ΔR^c_{11}, ΔR^s_{80}, ΔR^c_{80} represents the corrections (7) to the adopted ephemeris semi-diameter of the Sun $R_\odot = 959.63''$: . The part of $\Delta\sigma$ depends on corrections to adopted semi-diameters of the Sun $\Delta R_\odot$ and Mercury ΔR, and it is described by the relation:

$$\Delta\sigma_r = \frac{\Delta R_\odot}{\rho_\odot} \pm \frac{\Delta R}{\rho}, \qquad (9)$$

where the upperand low sign refer to the external or internal contacts correspondingly. Transforming the geocentric values $\Delta\sigma_r$ into the heliocentric ones Δc by multiplication on ρ/r and neglecting the correction to well-known semi-diameter of Mercury R =2639.7 км (or $3.37''$ at unit distance), we obtain the coefficient for $\Delta R_\odot$, coinciding with the Newcomb's expression [31] $(r^-1 - r^-_\odot 1)$. Here $r_\odot$, r are heliocentric distances of the Earth and Mercury correspondingly (more strictly it is the coefficient for linear combination of corrections to radii $\Delta R_\odot \pm 1.60\Delta R$).

$\Delta R_\odot$ includs the periodical corrections to the solar radius ΔR^s_{11}, ΔR^c_{11}. They describe a possible variation of the radius during the 11-cycle of activity. Variable $\varphi_{11} = (t - t^{(1)}_{min})/(t^{(2)}_{min} - t^{(1)}_{min})$ describes a phase of cycle: $t^{(1)}_{min}$ is the date of the nearest sunspot minimum, preceding to the date of the observation, and $t^{(2)}_{min}$ is the nearest next sunspot minimum. This form of the phase is connected with the variable length of the 11-cycle which may vary from 8 up to 15 years. The values of data of the minimums were taken from [44]. A power of the Sun activity (the number of sunspots at maximum of $W_{\odot,m}$) also varies for different cycles, so the simple model of coupling of $\Delta R_\odot$ and $W_{\odot,m}$ was used. It was supposed that the amplitude of variation of R_{11} is proportional to the number of sunspots in the maximum at the given cycle $W_{\odot,m}$. The mean value $W_{\odot,m}$ over the all observational interval was chosen as normalizing factor, equal 100. Such form of the unknown for 11-cycle permits to reduce all values of ΔR_{11} for given data of observations to the standard system of phases with the standard value of the Wolf's number at maximum. It increases probability to detect the possible variation of the solar radius more reliably. Moreover the unknowns ΔR_{80} were also included into equations. They dealt with 80 years periodicity of $R_\odot$, which was detected by Gilliland [7] and Shapiro [10]. The phase of the 80-cycle was calculated as $\varphi_{80} = (t - 1919.0)/80$.

Uniform ephemeris time scale ET is determined with deviations of observed positions of the Moon from calculated without taking into account of tidal effects. The corresponding correction equals $\Delta T^s = +1.821(\dot{n}_m/2)T^2$, where $\dot{n}_m = -26''/cy^2$ is adopted value of the lunar tidal deceleration, and the coefficient $1.821[s/'']$ is inverse value of mean motion of the Moon per second. The values of ΔT^s, based on analysis of observations of lunar occultations of stars, eclipses of the Sun, and transits of Mercury [56], are taken from [?]. The variation of the distance between the centers of the Sun and Mercury is $\dot{D}\Delta T$. As stated above this member is included also into the right hand of (6). Certainly the correction k to the adopted value of $\dot{n}_m$ must be included in the equation. Formally the term ($0.911(\rho/r)\dot{D}kT^2$) also enters in the left hand of (6). However the lunar tidal deceleration is determined by lunar laser ranging with very high accuracy, so k couldn't take into account (this unknown was used only at some cases for control of calculation).

4. Discussion of results

The various systems of weights were applied for processing of observations. A relative (partial) weight p changing from 0 to 1 was assigned depending on conditions and accuracy of the observation of the given transit and contact. For the observations made with screen relative weights equal to 0.3 were assigned . Then all observations of the same contact were weighted by mytiplication their partial weights on the factor that accounts the common circumstances of all observations. For instance the May transits give more precise heliocentric position of Mercury then the November transits about two times, as $(\dot{D}_V < \dot{D}_{XI})$.

The solutions of various sets of observations and the various weight systems are given in the table 4 (solution II is the one with equal weight). It is seen that the solutions are stable with respect to any chosen weight system, except for the parameters N, $\dot{N}$, connected with the correction to the longitude of node of Mercury's orbit. Parameters of 11-year and 80-year variations of the solar radius are determined reliable.

Influence of the time span of observations is shown also in tabl.4. The estimations of all parameters for the solutions Ia, Ib do not change, except the corrections $\Delta R^o_\odot$ и $\dot{R}_\odot$. It is clear that the noticable secular decrease of the solar radius (Sol.Ia) is caused by large difference of $\Delta R_\odot$-corrections before 1700 and after 1950 (see the table 5). It could be explained by a large number of observations, made after 1950 by amateurs used screen with small telescopes. At these observations the moments of internal contacts are removed systematically to the timing of the middle of the event. As a result the observed semi-diameter of the Sun decreases. (For the 17th century the causes could be observations made with screen and poor quality telescopes). After rejection of all screen observations (tabl.4, Sol.Ic) the module of $\dot{R}_\odot$ is diminished more then twice.

The obtained periodic corrections to the solar radius ΔR_{11} and ΔR_{80} are stable enough as for various weight systems, so for various time intervals. Thus we have the following expressions for variation of the solar radius (in seconds of arc):

$$\Delta R''_\odot = \frac{W_{\odot,m}}{100} \left[(0.15 \pm 0.02)\sin(2\pi\varphi_{11}) - (0.06 \pm 0.02)\cos(2\pi\varphi_{11})\right] \quad \text{without determination of 80-year variation,} \qquad (10a)$$

$$\Delta R''_\odot = \frac{W_{\odot,m}}{100} \left[(0.08 \pm 0.02)\sin(2\pi\varphi_{11}) + (0.02 \pm 0.02)\cos(2\pi\varphi_{11})\right] + (-0.18 \pm 0.03)\sin(2\pi\varphi_{80}) + (0.16 \pm 0.03)\cos(2\pi\varphi_{80}) \quad \text{with determination of 80-years variation,} \qquad (10b)$$

where $\varphi_{11} = (t - t^{(1)}_{min})/(t^{(2)}_{min} - t^{(1)}_{min})$, $\qquad \varphi_{80} = (t - 1919.0)/80.$

Table 4. The dependence of solution from method of observation and system of weights (in 0.01″).

Unknown	Sol.Ia	Sol.Ib	Sol.Ic	Sol.I	Sol.II	Sol.III
	1677– –1973 all obs.	1720– –1940 all obs.	1677– –1973 without screen	1677– –1973 without screen	1677– –1973 without screen	1677– –1973 without screen
	Weight of observations:					
	$p \cdot \dot{D}^{-1}$	$p \cdot \dot{D}^{-1}$	$p \cdot \dot{D}^{-1}$	$p \cdot \dot{D}^{-1}$	$p = 1$	p
V	$+47 \pm 6$	$+49 \pm 7$	$+48 \pm 6$	$+53 \pm 6$	$+40 \pm 6$	$+40 \pm 5$
$\dot{V}$	-72 ± 10	-80 ± 12	-71 ± 11	-62 ± 11	-56 ± 10	-56 ± 10
W	$+28 \pm 5$	$+31 \pm 6$	$+29 \pm 5$	$+28 \pm 5$	$+26 \pm 6$	$+27 \pm 6$
$\dot{W}$	-32 ± 8	-28 ± 11	-31 ± 8	-35 ± 9	-34 ± 10	-39 ± 9
N	$+12 \pm 5$	$+12 \pm 8$	$+1 \pm 5$	-13 ± 5	-15 ± 7	-10 ± 6
$\dot{N}$	$+54 \pm 5$	$+36 \pm 12$	$+27 \pm 8$	$+14 \pm 8$	$+4 \pm 12$	$+16 \pm 9$
$\Delta R^o_\odot$	$+8 \pm 2$	$+17 \pm 2$	$+12 \pm 2$	$+16 \pm 2$	$+15 \pm 2$	$+15 \pm 2$
$\dot{R}_\odot$	-16 ± 3	$+4 \pm 4$	-6 ± 3	-6 ± 3	-8 ± 4	-8 ± 4
ΔR^s_{11}	$+13 \pm 2$	$+14 \pm 2$	$+15 \pm 2$	$+8 \pm 2$	$+10 \pm 2$	$+9 \pm 2$
ΔR^c_{11}	-6 ± 2	$+0 \pm 2$	-6 ± 2	$+2 \pm 2$	$+2 \pm 2$	$+1 \pm 2$
ΔR^s_{80}	–	–	–	-18 ± 3	-15 ± 3	-14 ± 2
ΔR^c_{80}	–	–	–	$+16 \pm 3$	$+17 \pm 3$	$+17 \pm 2$
σ''_o	1.188″	1.194″	1.192″	1.127″	1.452″	1.342″

Table 5. The correction $\Delta R_\odot$ from 50-years intervals (in 0.01″).

Period	$\Delta R_\odot$ with screen	$\Delta R_\odot$ without screen
< 1700	-44 ± 44	–
1700 – 1750	+34 10	$+23 \pm$ 12
1750 – 1800	– 3 5	– 4 5
1800 – 1850	+26 6	+27 6
1850 – 1900	+14 3	+12 4
1900 – 1950	+24 3	+27 3
> 1950	–15 3	– 5 47
$\dot{R}_\odot$	–16 26	– 6 3

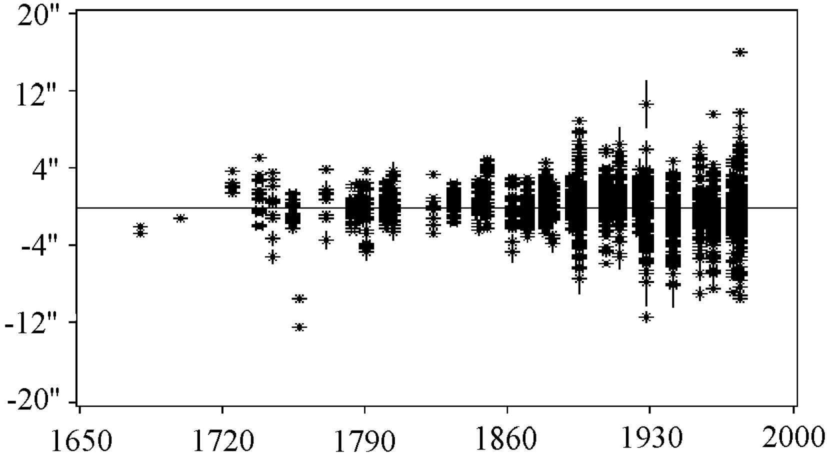

Fig. 7. Initial (O–C) of moments of internal contacts, expressed in angular units: $\sigma'' = [(O-C)^s + \Delta T^s] \cdot \dot{D}$. Here σ'' is a distance between the limbs in the ephemeris moment of contact; ΔT^s is a correction $ET - UT$; $\dot{D}''/s$ is the rate of approaching of the centers.

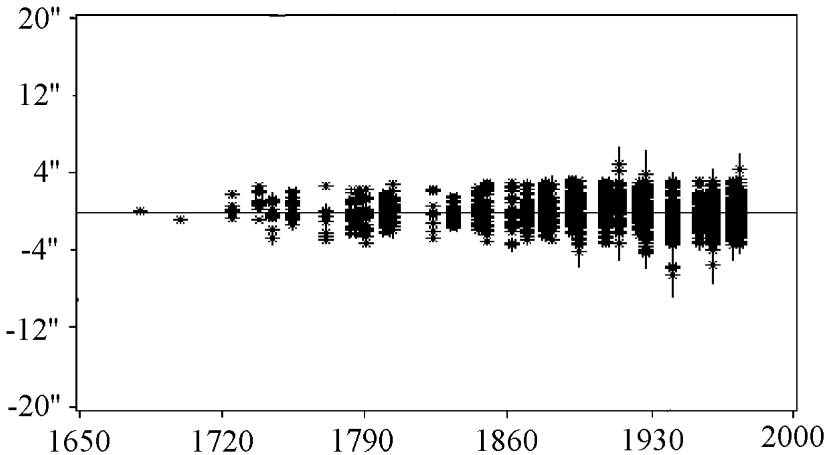

Fig. 8. Residuals of moments of internal contacts, expressed in angular units (for the solution Sol.I).

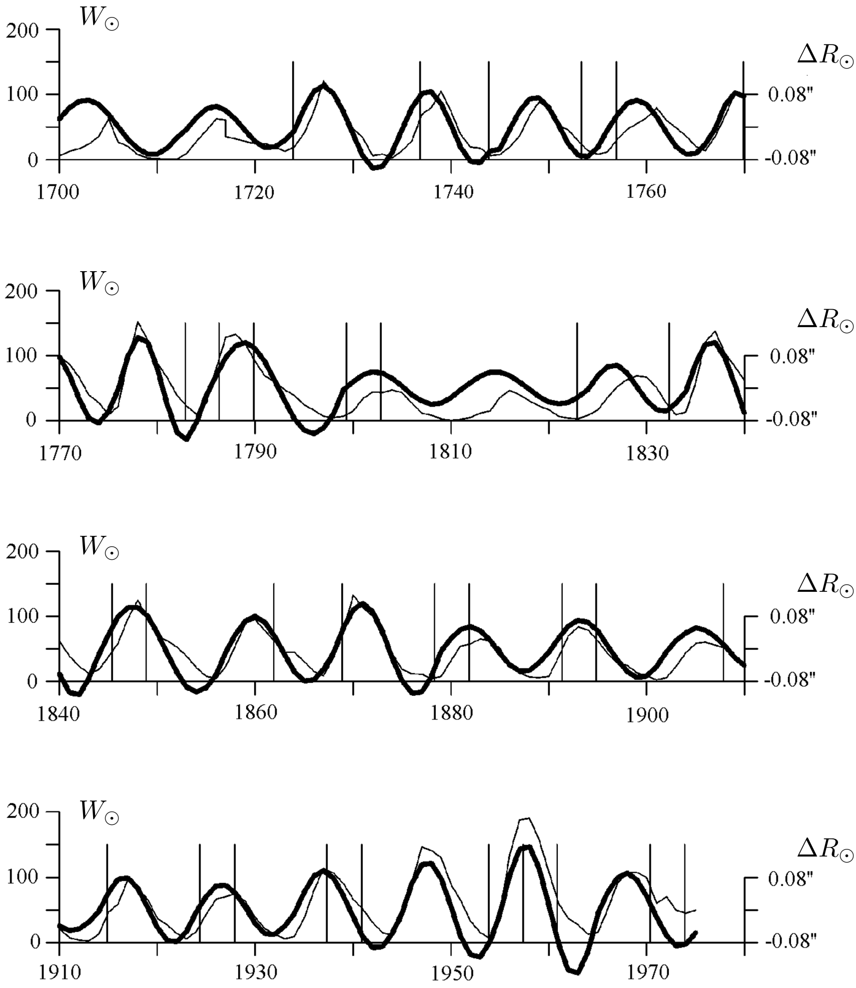

Fig. 9. Comparison between the 11-year variation of the solar radius ΔR_{11} (thick curve) and mean annual Wolf numbers (thin curve) for the solution Sol.I. The vertical segments are the data of observed transits of Mercury.

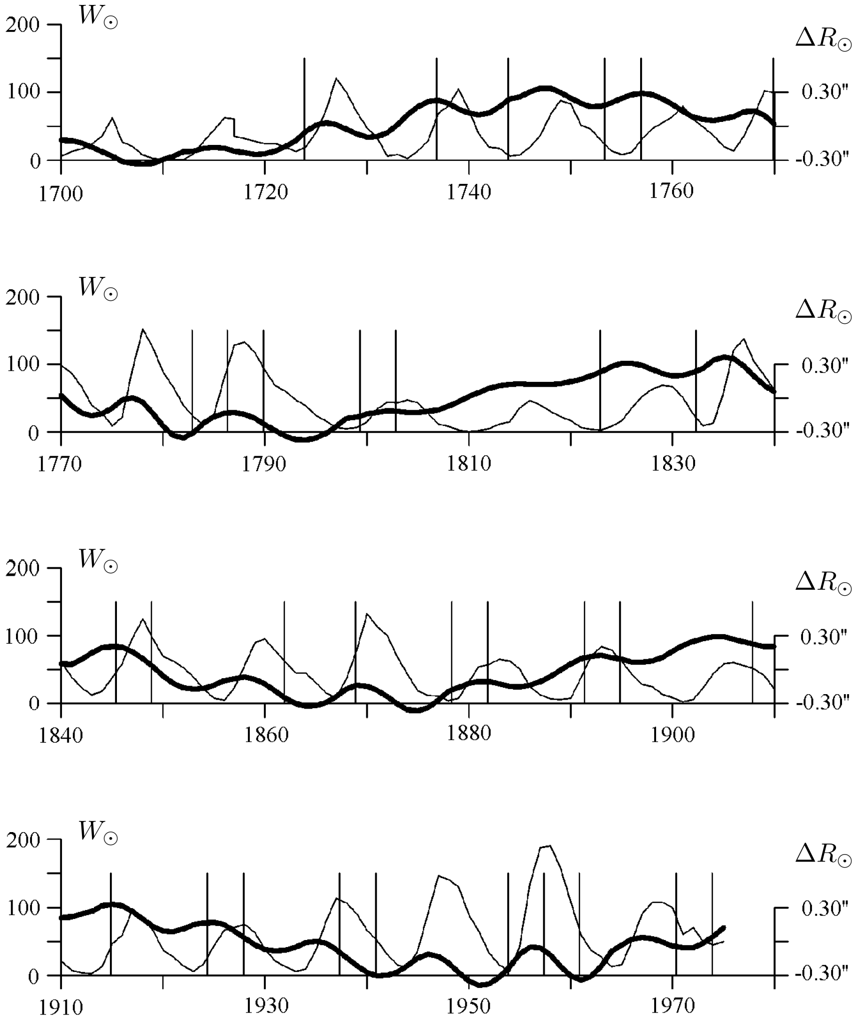

Fig. 10. Joint action of the 11-year and 80-year variations of the solar radius $\Delta R_\odot$ (thick curve) and mean annual Wolf numbers (thin curve) for the solution Sol.I. The vertical segments are the data of observed transits of Mercury.

Comparison between the 11-year variation of the solar radius ΔR_{11} and mean annual Wolf numbers are given in fig. 9. Joint action of the 11-year and 80-year variations of the solar raduis $\Delta R_{\odot}$ and mean annual values $W_{\odot}$ are shown in fig. 10.

The obtained result for the 80-year variation of the Sun's radius agrees with Shapiro [10] and Gilliland [7] as for the amplitude as for the phase.

It is more interesting that the curves of ΔR_{11} and $W_{\odot}$ show a good fitting of the phases. Some disagreement could be explained by different forms of curves during the 11-cycle (fig. 11): ΔR_{11} is represented by a sinusoid, but the curve of $W_{\odot}$ is asymmetric with the shifting maximum, which phase depends on value $W_{\odot}$ in maximum for. (It should be noted that variation of Wolf number from the phase φ is well described by the relation $W_{\odot} \simeq a\varphi^b e^{-c\varphi}$, where a, b, c are some constants [15]). The difference of curves is significant for cycles with a small value $W_{\odot,m}$. The correlation between the curves of ΔR_{11} and $W_{\odot}$ equals $+0.7 \pm 0.1$.

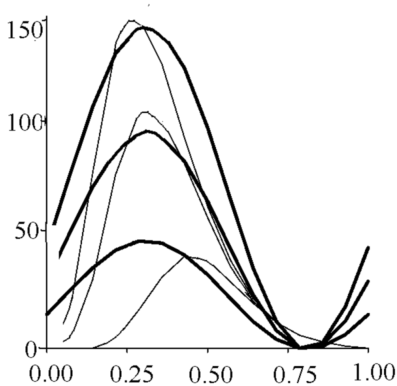

Fig. 11. Normalized curves of ΔR_{11} (thick ones) and $W_{\odot}$ with $W_{\odot,m}$ equal 150, 110, 45 during 11-cycles of the solar activity.

For a control of the values of ΔR_{11} obtained from observations up to 1973 a comparison is carried out with direct measurements of the Sun's semi-diameter after 1974. On fig. 12 it is shown: averaged month Wolf numbers, 11-year oscillations ΔR_{11} of solution Sol.I (thick curve); 11-year variations of radius, detected from astrolabe measurements at South Alps by Laclare et al. [19] (thin curve). The mean values of the radius derived from some sets of meridian and astrolabe observations also are given as black squares. They are taken from summary tables of Laclare et al. [19], Noel [23], Toulmonde [14]. It is seen that the values of ΔR_{11} *obtained in our work, are almost in phase with Wolf number variations,* *but variations* *of the Sun's radius obtained by Laclare et al. is almost in opposite phase with $W_\odot$ variations.* Averaged curve presenting other measurements (the dashed curve) has the amplitude about $0.1'' - 0.2''$ and approximately agree in the phase with variations of $W_\odot$. It should be noted that Noel's measurements (astrolabe, Santjago) [23] coincide in phase with $W_\odot$, but different sharply from other observations as in the amplitude of variations, so in the absolute value of the solar radius (Noel observed with CCD-astrolabe as Laclare et al. [19] did).

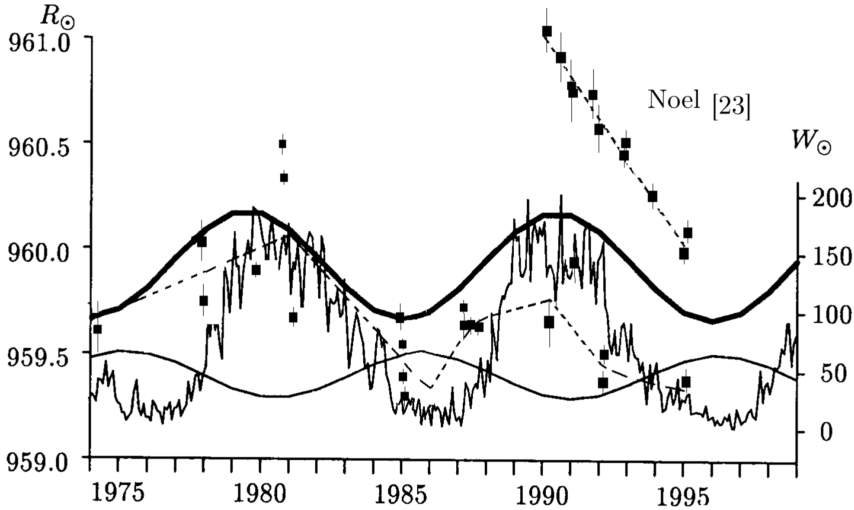

Fig. 12. Measurements of the solar radius during 1974–1998 (see the text).

Thus the observations of transits of Mercury, made in 1631–1973, give clear evidence that there exist *apparent* 11-year oscillations of the solar radius. Moreover there are the positive correlation between these oscillations and variations of $W_\odot$. However a problem arises whether the obtained apparent variations are the real oscillations of the solar radius or they are a result of some systematical errors of observations. Let us consider some possible sources of such errors.

1. Instrumental errors, connected with continual improving of quality of used telescopes during 300-year interval, could be source only of the secular component. For example it was shown by Toulmonde [14] for meridian measurement of the solar radius. In the case of transits such instrumental errors could be generated by the method of the screen. So it seems a very

probable that the obtained secular variation $\dot{R}_\odot \simeq 0.1''/cy$ is physically fictitious. The appearance of the 11-cycle at $\Delta R_\odot$ from such types of errors is a little probability.

2. Main period of repetition of transits is equal to 46 years, because a duration of 46 periods of revolutions of the Earth with respect to the nodes of Mercury is approximately equal to duration of 191 draconic periods of Mercury [33]. Due to small difference in durations of these time intervals the chord, drawn on the solar disk by Mercury, is shifted on $1.7'$ to the North for November transits and on $3.2'$ to the South for May ones (fig. 13). Within a 46-year period the relations between draconic and synodic periods of Mercury some additional periods appear: 7-year period with displacement of chord on $22.8'$ to the South; 13-year period ($8.2'$ to the North) and 33-year period ($6.5'$ to the South). May transits are often separated from November ones by 3-year period.

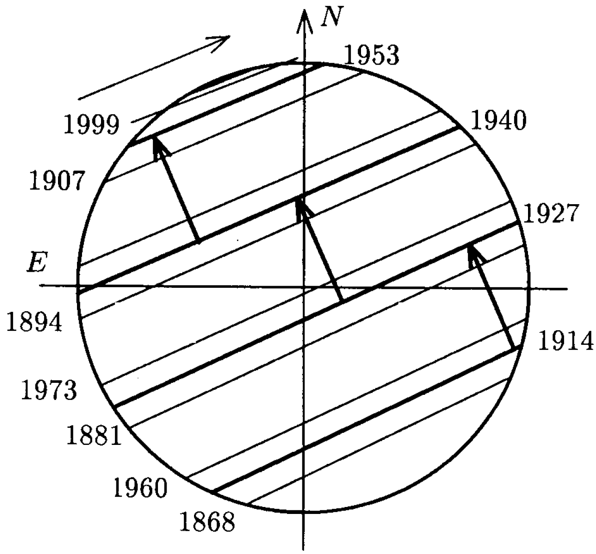

Fig. 13. The displacements of Mercury tracks on the Sun disk.

If noticeable apparent oblateness of the Sun do exist these displacements of tracks could give rise a periodic component of radius variations. However from helioseismic measurements of equipotential surfaces of the Sun the values of dynamic oblateness $J_{2\odot}$ were obtained as $(3.4 \pm 1.0) \times 10^{-6}$ [45] and $(2.18 \pm 0.06) \times 10^{-7}$ [46]. Analysis of planetary radar ranging gives $J_{2\odot} = (-0.13 \pm -0.41) \times 10^{-6}$ [47]. In the ephemerides DE405 was adopted the value $J_{2\odot} = 2 \times 10^{-7}$. It means that the difference between the equatorial and polar semi-diameters is less $0.01'' - 0.02''$. This estimation is affirmed by direct measurements of visible oblateness of the Sun, made at Pic-du-Midi [25] ($R_e - R_p = 0.011'' \pm 0.003''$) and by Dicke's data [48] ($R_e - R_p = 0.015'' \pm 0.002''$).

On the other hand the displacements of tracks change geometric conditions of observations and affect the determination of the apparent solar radius. For researches of the influence of shift of tracks or supposed oblateness of the Sun the transits were separated on the low-latitude transits, in which the minimal distances between centers was less than $\rho_{min} = 820''$, and the high-latitude ones. The results are represented in Tabl.6. It is seen that the amplitudes of oscillations are nearly equal, that the positive correlation with $W_\odot$ is remained in the both cases and runs up 0.6 and 0.8 for transits of equatorial and polar zones correspondingly.

Table 6. Influence of various factors on appearance of 11-year component into the Sun radius variation in form

$$\Delta R = A\sin(2\pi\varphi) + B\cos(2\pi\varphi) = C\sin(2\pi\varphi + \varphi_o).$$

N	Solution	A''	B''	C''	φ_o°
	The Sun's radius (direct measurements):				
1	Laclare [19]	+0.04	+0.10	(+0.11)	(+68°)
1a				+0.11	+135°
	The Sun's radius (Mercury's transits):				
2	all (Sol.I)	$+0.08 \pm 0.02$	$+0.02 \pm 0.02$	$+0.08 \pm 0.02$	$+11° \pm 12°$
3	low-latitude	$+0.08 \pm 0.03$	$+0.04 \pm 0.02$	$+0.09 \pm 0.02$	$+30° \pm 15°$
4	high-latitude	$+0.05 \pm 0.03$	-0.09 ± 0.04	$+0.10 \pm 0.04$	$-60° \pm 20°$
	Radii of the major planets:				
5	all methods	$+0.04 \pm 0.03$	$+0.00 \pm 0.03$	$+0.04 \pm 0.03$	$0° \pm 25°$
6	heliometer+ double image	$+0.04 \pm 0.03$	$+0.03 \pm 0.03$	$+0.05 \pm 0.03$	$+30° \pm 20°$

Note. In line n.1 the expression for $\Delta R_\odot$ is obtained with $\varphi = (t - 1985.15)/10.4$. The line n.1a gives Laclare's result, reduced to the epoch of phase t_o=1987.1, which is general for all rest of rows. With this value of t_o the variations of $W_\odot$ are described as $W_{\odot,m}\sin(2\pi\varphi - 90°)$.

3. Often used explanation of 11-years oscillation $\Delta R_\odot$ is a turbulence of the Earth's atmosphere, which properties depend on a phase of solar activity. In fact the term "the Sun's radius" is no rigorously defined and its value depends on method of observation or parameters of equipment used and seeing conditions [36]. Probably it is the main source of discripancy of the results (especially ofmeridian measurements). In particular the pecularities of local astroclimatic conditions could explain some contradiction of results obtained by different authors. From this point of view the observations of contacts are much objective.

Analysis of the atmospheric factor influence could be produced only by space astrometry methods. Nevertheless the approximate control is possible with direct measurements of planet semi-diameters. If such atmospheric influence exists then the planet radii measurements has to include 11-years oscillation as well as the solar radius. At present paper we used the results of measurements of the equatorial and polar radii of Mars and Jupiter and the equatorial radius of Saturn made in 1820–1950 [49], [50]. The determination of 11-year component of R was worked out in the same form as the solar radius as. Given in tabl.6 results show in some degree it is possible to explain 11-cycle in $R_\odot$ by atmospheric actions. However the obtained effect is very small, and so the influence of the atmosphere cannot be considered as proved fact for the 11-year component of $\Delta R_\odot$.

4. At last the apparent oscillations of the Sun's radius can be interpreted directly as oscillations of some layer of the solar photosphere. In this case the interpretaion also is not clear also. During observations (by eye or CCD-matrix) some isophotos is registered as an edge of the Sun. Obtained oscillations would be not a real oscillations of the solar radius, but a variation of distribution of isophotos on its disk from variation of temperature regime in surface photosphere layer during the solar activity cycle. The depth of this layer is about 100 km [51], and it corresponds to $\simeq 0.12''$ in angular units.

Further development of transit investigations should includ observations of contacts made in 1986–1999. The expansion of observational array would be realized with observations of "antitransits" that is occultations of Mercury by the Sun. These events could be registered more often twice then usual transits. On fig. 14 thick chords show paths of Mercury on the Sun disk from 2000 to 2026, and thin chords correspond to occultations of Mercury in the same period (the directions of Mercury motion are given by arrows). The table 7 contains the data of coming events (transits and occultations), the minimal distance of Mercury from the solar center ρ and the positional angles of first and fouth contacts P_1, P_4. The experimental observations of the Venus' occultation by the Sun were made in 1986 [54]. The main bulk of such observations is the determination of the solar limb position.

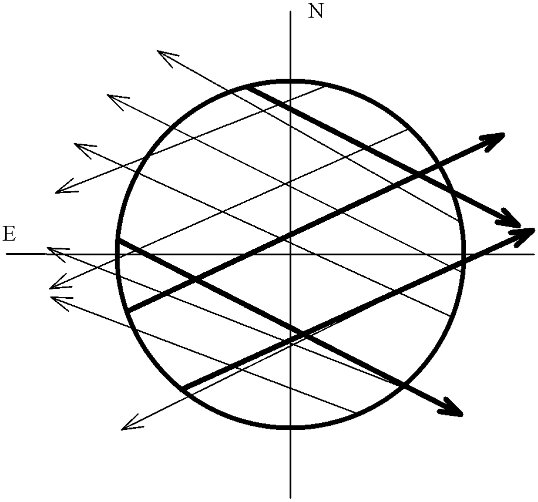

Fig. 14. Transits and occultations of Mercury in 2000–2026.

Table 7. Transits (tr) and occultations (oc) of Mercury in 2000–2026.

Date, Type	ρ''_{min}	P_1°, P_4°	Date, Type	ρ''_{min}	P_1°, P_4°
2000.05.09, oc	65	248, 62	2013.05.11, oc	297	263, 48
2002.11.14, oc	450	268, 135	2106.05.09, tr	320	84, 224
2003.05.07, tr	710	14, 290	2019.11.11, tr	76	109, 300
2006.11.08, tr	424	140, 275	2020.05.04, oc	394	220, 88
2007.05.03, oc	634	203, 100	2022.11.08, oc	320	316, 98
2009.11.05, oc	730	347, 75	2026.05.14, oc	534	280, 33

Special attention should be paid to observations of nearly tangent transits, which permit to estimate the polar radius value [52]. From them the apparent oblateness of the Sun $f_\odot$ could be determined. The dynamic oblateness of the Sun can be calculated from $f_\odot$ only for postulating of some model of inner structure of the Sun (in the simplest cases $J_{2\odot} = \frac{2}{3} f_\odot$ [53]). But this deficiency is inherent to interpretaion of any measurements of oblateness of apparent solar figure. It should be noted that the difference of values of $J_{2\odot}$, obtained from the apparent oblateness and from processing of radar ranging observations, can be additional infromational source about the inner solar structure.

5. Conclusions

In Institute of Applied Astronomy RAS an archive of observations of Mercury's transits through the solar disk has been collected. In the time interval 1631–1973 it includes about 4500 moments of contacts for 39 events, 2700 of them being the internal contacts.

We resume the main results obtained from analysis of Mercury's transit observations, covered more 25 cycles of the solar activity.

— The correction $+0.16'' \pm 0.02''$ is found to the ephemeris solar semi-diameter (i.e. $R_\odot = 959.79'' \pm 0.02''$ at the unit distance). It agrees with the estimation of Toulmonde [14] from 300-year set direct measurements (meridian, heliometer, astrolabe) of the radius. It agrees also with majority of modern measurements.

— It is shown the secular decrease of the Sun's radius $\dot{R}_\odot$ does not exceed $-0.06'' \pm 0.03''$, and it is caused by systematical errors of observations (particularly observations made with screen and by small telescopes).

— It is demonstrated that observations of moment of contacts are very informative for research of the solar radius variations, including the 11-year oscillations. The estimations of the 11- and 80-year periodic variations are stable with respect to various systems of weights.

— Obtained parameters of 11- and 80-year variations of the solar radius are following:

$$\Delta R''_\odot = \frac{W_{\odot,m}}{100}\left[(0.15 \pm 0.02)\sin(2\pi\varphi_{11}) - (0.06 \pm 0.02)\cos(2\pi\varphi_{11})\right]$$

without determination of 80-year variation,

$$\Delta R''_\odot = \frac{W_{\odot,m}}{100}\left[(0.08 \pm 0.02)\sin(2\pi\varphi_{11}) + (0.02 \pm 0.02)\cos(2\pi\varphi_{11})\right] + (-0.18 \pm 0.03)\sin(2\pi\varphi_{80}) + (0.16 \pm 0.03)\cos(2\pi\varphi_{80})$$

with determination of 80-years variation,

where $\varphi_{11} = (t - t^{(1)}_{min})/(t^{(2)}_{min} - t^{(1)}_{min})$, $\varphi_{80} = (t - 1919.0)/80$, and $W_{\odot,m}$ is the number of sunspots at maximum of cycle, in which the observations of transit was made.

— Amplitude and phase of 80-year variation R_{80} agree with results of Gilliland [7] and Shapiro [10].

— The 11-year component of apparent variation of the solar radius R_{11} has the amplitude $0.08'' \pm 0.02''$ and fits well with phase of Wolf's number variation. This conclusion is consistent with results of other authors. However obtained result contradicts to conclusion of Laclare et al. [19]: the values of amplitudes are in agreement, but the phases of oscillations are almost opposite (they differ by about $120°$).

— It is shown that observations of Mercury's transits as meridian or astrolabe observation as do not provide reliable physic interpretation of obtained variations of the Sun's radius. In this paper some sources of appearance of 11-year oscillations are considered.

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