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Estimating Masses of Asteroids

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М. В. Васильев, Г. А. Красинский, Е. В. Питьева, Э. И. Ягудина. Об оценивании масс астероидов.

Ключевые слова: массы астероидов, радарные наблюдения, эфемериды больших планет.

Выполнено сравнение масс астероидов, полученных динамическими методами (в основном, из взаимных возмущений пар астероидов при тесных сближениях) с оценками, выведенными астрофизическими методами. Показано, что астероиды с массами $> 10^{-12}$ масс Солнца $M_{\odot}$ влияют на орбиту Марса на уровне точности радарных измерений марсианских посадочных аппаратов (ПА) Viking-1,2 и Pathfinder. По-видимому, динамические и астрофизические методы оценивания масс астероидов дополняют друг друга: первые дают хорошие результаты для нескольких наиболее массивных астероидов, но на требуемом уровне точности оценки оказываются ненадежными для большого числа малых астероидов, в то время как астрофизические методы позволяют реально оценить массы около 2000 астероидов, но не могут быть применимы для самых больших из них. Предпринята попытка расширить список 300 астероидов, возмущения которых учитывались при построении эфемерид DE403/405. Тесты, в которых варьировалось общее число возмущающих астероидов (до 351), продемонстрировали влияние дополнительных астероидов на эфемериды больших планет, создаваемые в ИПА РАН совместным численным интегрированием уравнений движения планет и астероидов. Показано, что астрофизические массы некоторых из дополнительных астероидов слишком велики, и их включение в динамическую модель значительно ухудшает представление наблюдений марсианских ПА.

Суммарный возмущающий эффект остальных астероидов учитывался использованием модели кольца, расположенного в эклиптикальной плоскости, с равномерным распределением в нем вещества. Масса M кольца и его радиус R были включены в число улучшаемых параметров. Получена оценка $M \approx 518 \cdot 10^{-12} M_{\odot}$ (без 300 крупнейших астероидов) с неопределенностью 10%; как следствие, для общей массы астероидов главного пояса получено $M_{belt} \approx 1800 \cdot 10^{-12} M_{\odot}$. Для среднего радиуса кольца выведена оценка $R \approx 2.80$ AU с неопределенность 3%.

По выведенным массам 300 крупнейших астероидов найдены оценки двух параметров теоретического распределения астероидов (основанного на теории фрагментации) в предположении, что в этом наборе отсутствует эффект наблюдательной селекции. Данное распределение экстраполировано на астероиды с малыми массами и получена оценка $M_{belt} \approx 1700 \cdot 10^{-12} M_{\odot}$ для общей массы пояса астероидов, хорошо согласующаяся с динамической оценкой, приведенной выше.

Эти результаты позволили вывести выражение для оценки общего числа малых планет в любом единичном интервале абсолютных звездных величин H. Сравнение с наблюдаемым распределением показывает, что в настоящее время порядка 10% астероидов с $H < 14$ уже открыто (согласно выведенному распределению таких астероидов должно существовать около 100000).

В дополнении дается список выведенных масс для 357 астероидов, тестированных в процессе улучшения радарных наблюдений ПА.

G. A. Krasinsky, E. V. Pitjeva, M. V. Vasilyev, E. I. Yagudina. Estimating Masses of Asteroids.

Keywords: asteroid masses, ranging observations, ephemerides of major planets.

Comparison of masses of asteroids derived by dynamical methods (mainly from mutual perturbations of pairs of asteroids with close encounters) with estimates obtained by astrophysical methods is carried out. It is shown that the asteroids with masses $> 10^{-12}$ of the solar mass $M_{\odot}$ noticeably affect the orbit of Mars on the level of accuracy of measurements of ranging to the martian landers Viking-1,2 and Pathfinder. It appears that the dynamical methods based on the ground astrometry, and the astrophysical methods are complimentary: the first ones give satisfactory results only for several biggest asteroids but fail to reach the needed level of accuracy for a great number of small asteroids, while the astrophysical methods allow to estimate reliably the masses about 2000 small asteroids but cannot be applied to the biggest ones. An attempt is undertaken to extent the list of 300 perturbing asteroids accounted in the adopted DE403/DE405 ephemerides. Several tests were tried in which the total number of perturbing asteroids varied (up to 351) when processing the observations of the landers. In these tests perturbations from the asteroids are computed by simultaneous numerical integration of the equations of motion of the major planets and the asteroids. It has been revealed that the astrophysically estimated masses of some of these

additional asteroids are too large and their accounting in the dynamical model seriously deteriorates fitting to the lander measurements.

Overall perturbing effect of remaining asteroids has been modelled as being caused by a circular ring in the ecliptic plane. Mass M of the ring and its radius R are considered as solved-for parameters. The estimate $M \approx 518 \cdot 10^{-12} M_{\odot}$ is obtained (the 300 biggest asteroids are excluded) with the uncertainty 10%; as a sequence for the total mass of the main asteroid belt we have $M_{belt} \approx 1800 \cdot 10^{-12} M_{\odot}$. For the mean radius of the ring the estimate $R \approx 2.80$ AU is derived with the uncertainty 3%.

Two parameters of a theoretical distribution of asteroids with given masses (based on the fragmentation theory) are estimated by fitting to the derived set of masses of the biggest 300 asteroids (supposing that there is no effect of the observational selection in this set). The obtained distribution is extrapolated to small masses and the value $M_{belt} \approx 1700 \cdot 10^{-12} M_{\odot}$ for the total mass of the asteroid belt is obtained in a good accordance with our dynamical finding given above.

These results made it possible to derive an expression for estimating the total number of minor planets in any unit interval of absolute magnitudes H. The expression is compared with the observed distribution; the comparison shows that at present about 10% of the asteroids with the absolute magnitudes $H < 14$ are discovered (according to the derived distribution about 100000 of such asteroids are expected to exist).

In Appendix a list of the derived masses of 357 asteroids tested by the analysis of the lander ranging data is given.

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Contents

1. Introduction

At present serious problems arise in construction of planetary ephemerides which could match contemporary positional observations of the highest accuracy. These problems are due to the necessity to take into account the perturbations caused by the minor planets. Especially sensitive to these perturbations are the measurements of ranging to the martian landers Viking-1,2, and Pathfinder (with the typical error about 7 meters). On this level of accuracy the ephemeris of Mars is significantly affected by the perturbations from a large number of the minor planets (Williams [52]). In the ephemerides DE200 [43] the perturbations from three biggest minor planets (Ceres, Vesta, Pallas) were accounted. The experience showed that fitting of the ephemerides to the lander data mentioned above is nonetheless poor. In the more advanced ephemerides DE403 [44] and DE405 [45] the perturbations from additional 297 asteroids were taken into account by some simplified method and as a result the lander data were successfully processed. For further progress of fundamental planetary ephemerides it is important to have got the most reliable values of masses of the perturbing asteroids, estimate the influence of the asteroids which have not been yet accounted, and develop something more rigorous method of modelling of this dynamical effect. The lander ranging data allows to control how many of minor planets perturb significantly the orbit of Mars to be taken into account. It is easy to prove that the perturbations caused by a minor planet with the mass m may be characterized by the parameter δ:

$$\delta = a\frac{m}{M_\odot}\left(\frac{a}{\Delta}\right)^2,$$

where a is the semi-major axis of the perturbed planet, Δ is the nearest distance to the perturbed planet, and $M_\odot$ is the solar mass. From this expression one can see that minor planets from the main belt with $m/M_\odot \approx 10^{-11}$ perturb positions of Mars on the level of several meters and such perturbations must be accounted for. Moreover the large number of even smaller asteroids also may contribute significantly if their summarized mass is of the order $10^{-11}M_\odot$. So it is necessary to compute the perturbations from the asteroids with the masses at least $10^{-12}M_\odot$. Masses of the biggest minor planets must be estimated with the same accuracy.

There are two groups of methods that allow to evaluate the masses of asteroids. The first group is hereafter referred as the astrophysical one. These methods are based on measurements of the flux of radiation from the asteroid and on spectral observations which provide its spectral class. The most important factor that affects the flux is the radius of the asteroid, other factors may be modelled with the accuracy needed [32]. Having obtained the spectral class one can attribute to the asteroid a taxonomic type, then a corresponding density of the asteroid may be related to this type. Combining the radius and density, the mass of the asteroid is easily calculated.

An important information on asteroid radii is also provided by observations of occultations of stars by minor planets [30], and by radar observations of minor planets of the main belt (which become a routine procedure to be produced — at present 37 asteroids of the main belt are measured in this way, see [27]).

In the methods of the second group the mass of the asteroid has to be estimated from its perturbations upon the motion of some other celestial bodies. These methods can be applied in the following cases:

1. The perturbed body is another asteroid for which a close encounter with the perturbing body occurs; in this case a conventional ground based astrometry is used.
2. Very close encounters of some asteroids took place with a space probe of a specially designed space program; as a result more sophisticated and precise onboard observations allowed to derive very accurate and reliable estimates of masses of these asteroids.
3. Several biggest asteroids affect the motion of Mars so strongly that their masses can be estimated from analysis of ranging to the Martian landers.

Up to now masses of about 30 asteroids are obtained by the dynamical methods. Masses of the overwhelming body of other asteroids are too small to be determined by the dynamic method in which the ground based astrometry is used; however their total impact on the motion of Mars and Earth is not negligible. Fortunately nowadays the astrophysical methods are greatly improved and have provided a dataset from which masses may be estimated for about 2000 asteroids. Very important contribution to this problem was made after launching the dedicated satellite IRAS (Infra Red Astronomical Satellite) which measured the infrared fluxes from a large number of asteroids.

In the present work we compare the masses obtained by these two methods, derive a system of the masses that seems to be the most reliable.

Results of a number of numerical integrations of the major planets varying the total number of perturbing minor planets are discussed in order to estimate the level on which they affect the accuracy of ephemerides of the major planets.

2. Astrophysical estimations of masses

At present 3316 radii, obtained by the astrophysical method, are published in open NASA database SBN ("Small bodies node of the NASA Planetary Data System", see http://pdssbn.astro.umd.edu). This set includes both the IRAS data (1991 entries) and results of some ground observations. For the most cases a taxonomic code of Tholen [49] is also given which allows to estimate densities of 1098 asteroids. These data may be checked and appended by the radii and taxonomic codes for about 300 asteroids from the papers Bowell et al. [6], Tedesco et al. [48], Howell et al. [19], Xu et al. [54], and Barucci et al. [3]. We referred the Tholen's taxonomic codes to the three compositional taxonomic types making use of the compositional interpretation of the asteroid taxonomy types after Bell [5]. These types are Carbonic (the notation C), Sillicum (S), and Metallic (M). The adopted correspondence is given by Table 1.

Table 1. Correspondence of Tholen's classes with densities

Tholen's classes	C, D, P, T, B, G, F	S, K, Q, V, R, A, E	M
Composition type	C	S	M
A priory density	1.8	2.4	5.0
Revised density, [46]	1.29±0.06	2.71±0.04	5.29±0.53
Revised density, [34]	1.36±0.03	2.67±0.02	-
Revised density, this work	1.38±0.02	2.71±0.02	5.32±0.07

The a priory densities for each of the three types of asteroids were used to calculate the perturbing accelerations from selected 297 asteroids when constructing the ephemerides DE403/DE405 ([44], [45]). The densities have been revised in the precess of fitting of these ephemerides to observational data [46]. The densities given in the last two lines have been derived in a similar way from an independent analysis of practically the same observational data (measurements of distances to the landers and to surfaces of the inner planets) while constructing the numerical ephemerides EPM2000 [34] (the line 4) and from the present analysis based on the same program package (the line 5). The starting system of masses of the minor planets was that of DE405; these data were kindly provided to us by M. Standish (private communication).

The astrophysical method being applied to three biggest minor planets (Ceres, Pallas, and Vesta) gives wrong results presumably because these planets have more complicated internal structure and their mean densities cannot be restored reliably from their spectral classes. Fortunately it appears possible to derive accurate masses of these asteroids by the dynamical method in the process of fitting of the planetary ephemerides to the ranging observations.

At present with the available IRAS radii and Tholen's classes of the asteroids it seems possible to determine the masses for a larger set of the asteroids in order to compute the perturbations from these bodies more accurately than it has been in DE403/DE405. So in this paper an attempt has been undertaken to take into account perturbations from 351 asteroids making use of the published IRAS data. The results are controlled by comparison with the masses adopted in DE405, and by fitting corresponding planetary ephemerides to the lander ranging data and other measurements (see Section 4).

A priory estimates of the errors of IRAS based masses vary in the range from 10% (for large asteroids) to 30% (for small ones). In the following two cases the a priory estimate may be checked by comparison with the masses of the minor planets (433) Eros and (253) Mathilde derived by the space mission NEAR to these asteroids ([55], [56]). The NEAR results and those based on IRAS data are presented in Table 2:

Table 2. Masses of Eros and Mathilde in $10^{-12} M_{\odot}$

	(433) Eros	(253) Mathilde
NEAR	0.003362	0.051938
IRAS	0.0058	0.0698

The errors of NEAR masses may affect only the last decimals of the values given in Table 2. One can see that the supposed error of IRAS masses for small asteroids (about 30%) is confirmed by the comparison with the NEAR estimates.

The mass distribution derived from the IRAS data making use of our estimates of the densities (the last line of Table 1) is presented in Fig. 1. The asteroids are ordered as their masses diminish $(m_1 > m_2 > ... > m_N)$ and the consequent sum of the masses $M = \sum_1^N m_i$ in the units $10^{-12} M_\odot$ is depicted as a function of N. The arrows on the curve mark the mass of Ceres and the total mass of the asteroids accounted in DE405. One can see that a noticeable part of the asteroids which affect the motion of Mars is not yet taken into account.

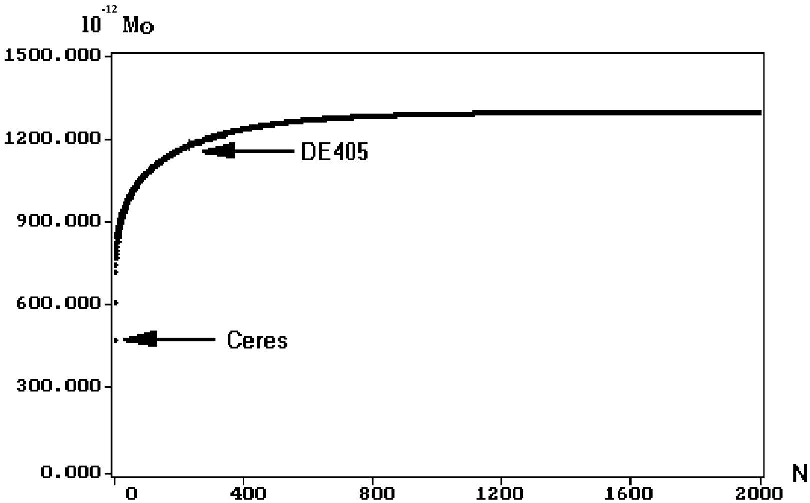

Figure 1. Summing masses derived from the IRAS data

3. Dynamical estimations of the asteroid masses

In the last years considerable efforts were devoted to determination of the masses of asteroids by the dynamical methods. It seems interesting to compare the obtained results with masses based on the IRAS estimations of asteroid radii.

Table 3. The masses of 26 asteroids in $10^{-12} M_{\odot}$ (dynamical estimations)

Asteroid perturbing	Asteroid perturbed	Num. obs.	Interval of obs.	Mass	σ
(1) Ceres	(348) May	185	1892-1998	477	15
	(2933) Amber	56	1920-1998	460	113
	(534) Nassovia	112	1904-1997	487	16
(2) Pallas	(2204) Lyyli	205	1940-1998	398	209
(3) Juno	(1346)Gotha	82	1929-1998	81	34
	(920) Rogeria	121	1919-1998	16	122
	(1767) Lampland	79	1941-1997	48	38
(4) Vesta	(2066)Palala	99	1931-1998	573	102
	(197) Arete	305	1906-1998	149	11
	(2873) Binzel	46	1935-1998	282	155
	(3002) Delasalle	80	1942-1998	90	27
(7) Iris	(836) Jole	90	1903-1998	-40	35
	(1825) Klare	86	1934-1998	11	10
	(571) Dulcinea	89	1905-1998	61	24
	(1007) Pawlowia	167	1906-1998	392	388
(10) Hygiea	(1259) Ogyalla	132	1900-1998	48	39
	(1780) Kippes	75	1906-1998	35	66
	(2619) Skalnate Pl.	61	1975-1998	198	46
	(465) Alekto	182	1901-1998	362	280
	(3946) Shor	72	1950-1999	-45	76
(15) Eunomia	(1284) Latvia	107	1925-1998	15	4
	(1313) Berna	85	1911-1998	9	1
(16) Psyche	(263) Dresda	213	1905-1998	50	32
	(2819) Ensor	74	1933-1998	59	44
	(2589) Daniel	71	1955-1998	197	25
(19)Fortuna	(46) Nestia	384	1904-1998	57	12
	(827) Wolfiana	163	1928-1998	29	28
(24) Themis	(2169) Taiwan	75	1965-1997	-18	34
	(2296) Kugultinov	71	1969-1997	11	5
	(2296)+4 aster.	468	1902-1996	10	4

Table 3. The masses of 26 asteroids in $10^{-12} M_\odot$ (continuation).

Asteroid perturbing	Asteroid perturbed	Num. obs.	Interval	Mass	σ
(31) Euphrosyne	(109) Felicitas	148	1907-1998	332	41
(45) Eugenia	(1055) Tynka	87	1902-1998	119	33
	(2560) Siegma	143	1932-1998	-43	14
	(2814) Vieira	148	1954-1998	-39	22
	(308) Polyxo	279	1902-1998	-9	9
(52) Europa	(1605) Milankovich	153	1907-1998	10	52
	(3019) Kulin	142	1940-1998	-57	20
	(1558) Jarnefel	79	1913-1994	-103	77
(65) Cybele	(147) Protogene	209	1902-1998	17	19
	(1624) Rabe	158	1928-1998	97	26
	(1668) Hanna	108	1932-1998	3	25
(87) Sylvia	(1461) Jean-Jacques	114	1935-1998	253	107
	(1081) Reseda	114	1927-1998	-22	10
	(2246) Bowell	92	1942-1998	140	60
(107) Camilla	(515) Athalia	113	1903-1998	13	99
	(1882) Rauma	51	1941-1998	16	25
(165) Loreley	(1737) Severny	89	1942-1998	299	55
	(1298) Nocturna	105	1904-1998	46	70
	(1913) Sekanina	120	1928-1997	-139	46
	(2964) Jaschek	54	1974-1998	4	194
(216) Kleopatra	(3976) 1983 JM	87	1943-1998	145	90
(324) Bamberga	(916) America	105	1924-1998	13	29
	(1240)Centenari	116	1915-1999	375	123
	(1066) Lobelia	121	1911-1997	174	78
(451) Patient	(977) Philippa	57	1914-1998	56	43
	(3286) Anatoliy	67	1978-1998	46	85
(511) Davida	(1847) Stobbe	105	1902-1998	317	43
	(4624) Stefani	78	1969-1998	18	150
(704) Interamnia	(881) Athene	129	1917-1999	68	67
(20) Massalia	(356) Liguria	169	1912-1994	13	7
(720) Bohlinia	(1029) La Plata	152	1916-1995	6	1
(804) Hispania	(1002) Olbersia	84	1930-1995	9	3
(1669) Dagmar	(2248) Kanda	107	1939-1996	21	16

The most part of the dynamical estimates are derived from analysis of mutual perturbations of pairs of asteroids having close encounters and based on the conventional astrometry. For three biggest asteroids (Ceres, Pallas, Vesta) the mass determination may be carried out also by analysis of weak perturbations produced by them upon the orbit of Mars making use of the measurements of ranging to the martian landers.

In our experiments of the mass determination by the method of close encounters (see Table 3) the set of the pairs of these encounters is taken from the published list of 31 such pairs (Hilton et all. [16]), which have been obtained by laborious numerical integrations of a large number of sampling pairs. We have selected 26 pairs of the asteroids for which the close encounters occurred before the year 2000 and so astrometric observations are available. As the first step the elements of the orbits both for perturbing and perturbed asteroids were improved, and then the conditional equations respectively to the mass under estimation were computed by numerical integration of variational equations. After evaluating the mass the process was iterated until convergence was reached.

Table 4 gives a summary of determinations by other authors (weighted values of our estimates given in Table 3 are also included), as well as the IRAS based estimates obtained by the method outlined in the previous section.

In Table 4 the symbol $\star$ marks the asteroids for which there are no data in the SBN database. Their masses are calculated using the radii and taxonomic classes used in DE405. They have been obtained by Williams [53] from IRAS and ground infrared data, occultations and photometric observations. The present study has confirmed that the set of the masses adopted in DE403/DE405 is of the excellent quality.

The main conclusion one can derive looking through Tables 3, 4 is the great superiority of the accuracy of the IRAS masses to that of the dynamical method based on the close encounters between asteroids. The IRAS masses prove to be accurate on the level $10^{-12} M_\odot$ while the typical errors of the best dynamical estimates are usually of one order greater. Masses of Bohlina and Dagmar are of the order $10^{-14} M_\odot$; such small masses cannot be detected by the dynamical methods based on ground astrometry.

Note that the masses of Ceres, Pallas and Vesta derived from ranging to the martian landers are consistent with those obtained by the close encounter method but have somewhat better accuracy.

Table 4. Summary of estimates of the asteroid masses in $10^{-12}M_{\odot}$

Perturbing asteroid	Perturbed body	Interval of obs.	Mass	σ	Reference
(1) Ceres	(2)Pallas	1803-1968	670	40	[38]
	(4)Vesta	1807-1968	510		[39]
	(2)Pallas	1802-1970	590	30	[40]
		1802-1983	499	9	[24]
		1802-1983	500		[36]
		1802-1987	521	30	[25]
	Mars		500	20	[42]
	(203) Pompeja	1879-1990	474	30	[13]
	(348) May	1892-1991	480	22	[51]
	(348) May	1892-1991	480	9	[41]
	6 asteroids	1892-1993	485	6	[7]
	(2) Pallas	1892-1993	510	20	[36]
	(4) Vesta	1802-1991	488	45	[16]
	5 asteroids	1879-1993	467	9	[10]
	Mars, DE403		464		[44]
	(203) Pompeja	1879-1992	426	9	[22]
	7 asteroids	1802-1991	471	5	[36]
	4 asteroids	1802-1996	492	7	[33]
	(3) Juno	1802-1991	469	27	[15]
	(4) Vesta	1802-1991	450	18	[15]
	(2) Pallas	1802-1991	437	7	[17]
	22 asteroids	1879-1998	484	8	[23]
	25 asteroids	1849-1998	470	4	[28]
	3 asteroids	1892-1998	479	11	this work
	Mars, EPM2000		481	1	[34]
	Mars		476	1	[46]
			476		adopted
(2) Pallas	(1) Ceres	1802-1970	130	30	[36]
	(1) Ceres	1801-1970	114		[40]
	(1) Ceres		108	30	[36]
	Mars		140	20	[42]
	Mars, DE403		105		[44]

Table 4. Summary of estimates of the asteroid masses in $10^{-12}M_\odot$ (continuation)

Perturbing asteroid	Perturbed body	Interval of obs.	Mass	σ	Reference
	(1) Ceres	1839-1992	157	6	[16]
	2 asteroids		121	26	[28]
	Mars, EPM2000		100	1	[34]
	Mars		108	1	[46]
			108		adopted
(4) Vesta	(197) Arete	1879-1962	138	6	[36]
	Mars		150	3	[42]
	Mars, DE403		134		[44]
	(197) Arete	1879-1992	158	11	[16]
	(1) Ceres	1839-1992	152	15	[16]
	22 asteroids		134	9	[23]
	26 asteroids		136	5	[28]
	4 asteroids	1906-1998	146	10	this work
	Mars, EPM2000		136	1	[34]
	Mars		135	1	[46]
			135		adopted
(3) Juno	22 asteroids	1919-1990	461	88	[23]
			10	1	IRAS
(6) Hebe	2 asteroids	1883-1997	7	2	[29]
			5	0.2	IRAS
(7) Iris	22 asteroids	1903-1998	122	58	[23]
			6	1	IRAS
(10) Hygiea	(829) Academia	1914-1978	47	23	[37]
	(829) Academia	1914-1990	21	13	[13]
	22 asteroids		63	14	[23]
	7 asteroids		57	14	[20]
	8 asteroids	1848-1998	56	7	[29]
	(1259) Ogyalia	1914-1998	48	39	this work
			28	1	IRAS
(15) Eunomia	(1313) Berna	1925-1991	4	1	[17]
	(1284)Latvia	1911-1991	2	7	[17]
	22 asteroids		8	4	[23]
	3 asteroids	1848-1998	13	3	[29]

Table 4. Summary of estimates of the asteroid masses in $10^{-12} M_{\odot}$ (continuation)

Perturbing asteroid	Perturbed body	Interval of obs.	Mass	σ	Reference
	2 asteroids	1911-1998	13	3	this work
			14	2	IRAS
(16) Psyche	3 asteroids	1905-1998	127	18	this work
	22 asteroids		-737	28	[23]
			25	1	IRAS
(19)Fortuna*	2 asteroids	1904-1998	53	11	this work
			3		DE405
(20) Massalia	(356) Liguria	1912-1994	13	7	this work
			2	0.5	IRAS
(24) Themis*	(2296)Kugultinov	1911-1995	29	13	[26]
			17	6	[23]
	5 asteroids	1902-1996	10	4	this work
			3		DE405
(31) Euphros.	(109) Felicitas	1907-1998	332	41	this work
			6	0.9	IRAS
(45) Eugenia	4 asteroids	1902-1998	-15	18	this work
			4	0.2	IRAS
(52) Europa	13 asteroids		29	16	[20]
	3 asteroids	1907-1998	-64	16	this work
	4 asteroids	1865-1998	26	9	[29]
			11	0.6	IRAS
(65) Cybele	7 asteroids		8	4	[20]
	3 asteroids	1902-1998	39	13	this work
			5	0.3	IRAS
(87) Sylvia	3 asteroids	1927-1998	27	10	this work
			7	1	IRAS
(88) Thisbe	(7) Iris	1848-1997	7	1	[29]
			4	0.3	IRAS
(107) Camilla	2 asteroids	1903-1998	16	24	this work
			5	1	IRAS
(165) Loreley	4 asteroids	1904-1998	41	31	this work
			1	0.1	IRAS

Table 4. Summary of estimates of the asteroid masses in $10^{-12}M_{\odot}$ (continuation)

Perturbing asteroid	Perturbed body	Interval of obs.	Mass	σ	Reference
(324) Bamberga	3 asteroids	1911-1998	49	30	this work
			5	0.5	IRAS
(444) Gyptis	(54) Alexandra	1863-1998	4	2	[29]
			2	0.4	IRAS
(451) Patientia	2 asteroids	1914-1998	54	38	this work
			4	0.3	IRAS
(511) Davida	2 asteroids	1866-1998	33	3	[29]
	2 asteroids	1902-1998	294	43	this work
			14	0.7	IRAS
(704) Interam.	2 asteroids	1923-1991	37	170	[36]
	22 asteroids		-6	83	[23]
	3 asteroids	1856-1998	35	9	[29]
	(881) Athene	1014-1998	68	67	this work
			13	0.7	IRAS
(720) Bohlinia	(1029)La Plata	1916-1995	6	1	this work
			0.03	0.004	IRAS
(804) Hispania	(1002) Olbersia	1933-1991	5	4	[50]
	(1002) Olbersia	1933-1998	9	3	this work
			1	0.2	IRAS
(1669) Dagmar	(2248) Kanda	1939-1996	21	16	this work
			0.02	0.004	IRAS

We underline once more that only the estimates of the masses with errors about $10^{-12}M_{\odot}$ are of a practical significance. Our experience shows that at present the comparable accuracy is reached with the use of the conventional astrometry only in a few favorable cases when several close encounters take place, or if ranging observations of the perturbed asteroids are available. Apart of the results for three biggest asteroids, in fact only for (15) Eunomia (and to some extent for (10) Hygiea) the results may be considered satisfactory being more or less consistent with the IRAS estimates. Nonetheless it may be hoped that very careful analysis of the ground astrometric data may provide the accuracy of asteroid masses compared with that of IRAS estimates (see, for instance, [28], [29]) where some hopeful results are presented).

It is important that the IRAS masses are available for 1991 asteroids while the less accurate masses derived from encounters of asteroids are obtained at present only for several dozen asteroids.

4. Perturbations from minor planets upon Mars

In this section we describe a number of experiments with integration of equations of motion of the major planets taking into account the perturbations from several sets of minor planets with masses estimated in different ways. We used the lunar-planetary integrator embedded in the program package ERA [21]. The integrator makes it possible to integrate simultaneously barycentric equations of motions of the nine major planets, Sun and Moon, equations of lunar physical libration, and equations of several hundred minor planets. For five biggest asteroids their mutual perturbations are accounted, for the other asteroids such perturbations are neglected. Due to some restrictions of the current version of ERA the maximal quantity of the minor planets to be integrated must not exceed 351. Values of astronomical constants involved were taken from DE405. Two versions of the initial state vectors for the major planets are considered. In the first version they have been obtained by fitting the integrated coordinates and velocities to those of DE403 on the time interval 1886–2011. In the second version [35] they have been derived as a result of fitting to all radiometric observations 1961–1997 presented in the website of Commission 4 IAU (http:/ssd.jpl.nasa.gov/iau-comm4/; the cited above JPL Interoffice Memoranda [44], [45] as well as the paper [35] also may be found in this site). For our aims the measurements of ranging to martian landers Viking-1, Viking-2, and Pathfinder are of paramount importance. This dataset was complemented with Russian measurements of ranging to the inner planets (1961–1995). Parameters of the lunar theory and the lunar physical librations were derived by processing LLR observations 1970-2000 which are also presented in cited website. An earlier version of this analysis was described in [1].

When dealing with the version fitted to DE403 it was noticed that the differences of the elements of Mars in the two theories under considerations arise mainly due to different methods of computing perturbations from minor planets; probably they may be explained by some simplifications made in DE403. Namely, in order to compute the coordinates of 297 minor planets in DE403 some averaged elements of these planets are used;

coordinates of three biggest asteroids are obtained by rigorous numerical integrations on one orbital period and then periodically extrapolated to the current date of the integration. In DE405 such type of extrapolation is applied to all 300 asteroids. In the both ephemerides the perturbing accelerations from the 297 asteroids (apart of three biggest ones) have been computed in advance and saved as a file making use of the a priory known coordinates of the major planets. In the process of numerical integration of the major planet equations the perturbing accelerations in need are retrieved from this file ([18], [37]). It has been verified that such simplified method is responsible for the most part of the detected differences of the planetary coordinates in our model with those in DE403/DE405. In more details these differences have been studied in [35]; a characteristic pattern of the biases in the semi-major axis and the mean longitude of Mars (in the sense DE403 minus our ephemerides) are presented in Fig. 2 and Fig. 3 for the time interval 1954–2001. When comparison is made with DE405 the time variations of the differences are practically the same. It appears that the irregular behavior of the differences is due to the close encounters of Pallas with Mars that occur at 1964 and 1973 are not modelled in the simplified approach adopted in the DE403/405 ephemerides. In the plots one can see jumps of 20 m in the semi-major axis and 0.3 mas in the mean longitude at the times of the encounters.

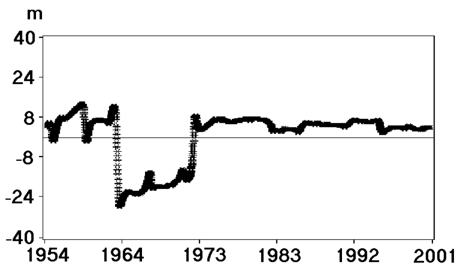

Figure 2. DE403 minus EPM, for the semi-major axis of Mars.

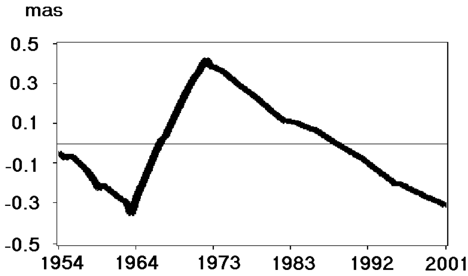

Figure 3. DE403 minus EPM, for the mean longitude of Mars.

As it was mentioned above the astrophysically derived masses have allowed to improve considerably the accuracy of the planetary ephemerides. Unfortunately adding new perturbing asteroids in order to reach further progress appears to be not a routine work because some problems are met as our experiments have shown.

In fact the SBN database keeps two set of radii: a set derived from IRAS observations (they are referred hereafter as the radiometric radii or the system 1 of radii) and a larger set based also on some ground observations (the system 2). In the SBN there is no reference concerning the first set; some information about the second one is given in [47]. For masses of 12 asteroids only the DE405 data could be used because these asteroids absent in the SBN database. It is easy to see that the radiometric radii of the system 1 are systematically less than those of the system 2; it also appears that in several cases the radii in the system 2 are roughly erroneous. The radiometric radii are supposedly more accurate, however they may need some calibration. Because in our analysis the densities of the asteroids are considered as solved-for parameters, the corresponding systematic error of the radii seems to be eliminated.

So it was naturally to suppose that the radiometric radii are preferable to use as the more reliable ones. However our considerations show that the system of the masses of 297 asteroids adopted in DE403/DE405 is more consistent with the system 2. The following four different sets of the masses of the perturbing asteroids were tried in our analysis of the ranging measurements mentioned above:

1. Solution 1: 300 asteroids with the masses taken from DE405.
2. Solution 2: 300 asteroids with the masses estimated independently from the radiometric dataset of the system 1. For 16 asteroids which

have no radiometric radii in the SBN database, the masses again have been taken from DE405.

3. Solution 3: 300 asteroids plus next largest 51 asteroids of the main belt for which the radiometric radii are given in the SBN database (the system 1).
4. Solution 4: 300 asteroids plus next largest 51 asteroids of the main belt with any SBN available radii (Chiron included). This set includes a number of asteroids for which there are no radiometric radii, only those given by the system 2. The total mass of the additional asteroids exceeds that in the solution 3.

In each solution the complete set of ranging observations has been processed, corrections to the densities of the three classes (C, S, M) being added to the list of the solved-for parameters. The results are presented in Table 5 as solutions 1–4 (in this table N means the solution number). The lines 1–3 present rms (in meters) of the postfit residuals for the ranging observations of Viking-1, Viking-2, and Pathfinder. In the next six lines the estimated densities ρ_C, ρ_S, ρ_M of three classes of asteroids and the corresponding errors are given. Note the catastrophic degradation of fitting of the solution 4, with the negative densities needed to reach even such poor fitting. It means that the masses of the additional 51 asteroids (at least the biggest of them) are erroneous being too large (see the next to last line of Table 5 where the total mass of the accounted asteroids is given). The set includes 41 considerably big asteroids for which the radiometric radii are not available. More satisfactory results are obtained for another set of 51 additional asteroids with the available radiometric radii (solution 3). The total mass of the asteroids of this set is considerably less. However in this case we also notice some deterioration of fitting of the ranging to Pathfinder in comparison with the best fitting that corresponds to the solution 1 (the masses from DE405). The solution 2 (300 asteroids with radiometric masses) is marginally worse than the solution 1 and the derived densities give somewhat worse accordance with the NEAR densities for (433) Eros and (253) Mathilde [55], [56].

Because the estimated densities for the classes C and S in the solution 2 exceed those of the solution 1, we can conclude that the radiometric radii are indeed smaller than their correct values which probably are near to the radii of the system 2 (if they are not spoiled by rough errors that is sometimes the case). So one has to be very cautious when selecting radii from the SBN database. It is desirable to make comparison of the both types of radii in order to reject the values when they are roughly erroneous. Inspecting the two system of radii in the SBN the following expression has been derived that allows to transform the radiometric radius R_r (in kilometers) of an asteroid to its value R corrected for the systematic error:

$$R = 1.0352R_r + 0.313$$

Table 5. Impact of minor planets on planetary ephemerides

N	1	2	3	4	5	6	7	8
σ_{Vik1}	7.9	8.5	8.2	24.8	7.5	7.8	8.2	7.5
σ_{Vik2}	5.6	5.7	5.7	14.7	5.5	5.4	5.5	5.7
σ_{Path}	3.1	3.4	8.5	60.2	4.0	3.0	8.5	3.6
ρ_C	1.38	1.45	1.19	-1.60	1.54	1.34	1.12	1.74
	±0.03	±0.03	±0.02	±0.04	±0.07	±0.04	±0.04	±0.11
ρ_S	2.71	3.22	2.84	0.76	2.71	2.71	2.82	2.64
	±0.02	±0.02	±0.02	±0.02	±0.03	±0.02	±0.03	±0.08
ρ_M	5.32	3.90	5.63	-5.29	6.60	5.30	5.35	4.21
	±0.07	±0.08	±0.20	±0.29	±0.43	±0.13	±0.30	±1.35
R_{ring}						2.94	2.83	2.77
						±0.06	±0.06	±0.06
M_{ring}						529.4	397.6	479.7
						± 53.5	±54.2	±55.4
N	300	300	351	351	351	300	351	351
M_{tot}	1177	1127	1198	1560	1477	1177	1198	1493
M_{sum}						1706	1596	1973

It might be thought that the deterioration of fitting in the 4-th solution is caused by an error of the mass derived from the SBN radius R=419 km of the asteroid (2060) Chiron which contributed to this solution. However the semi-major axis of the orbit of Chiron is rather large and it is the reason why our analysis has shown that Chiron only intangibly affects the orbit of Mars. Indeed deleting this planet from the list of the perturbing asteroids practically does not change the residuals for the martian landers. Nonetheless it appears that Chiron had close encounters with Uranus and Saturn and as a result the perturbations in the mean longitudes of these planets reach 0.7 mas and 10 mas. For Jupiter the perturbations reach 3 mas which correspond to variation about 20 km in the distance from the Earth. Recent estimates of the radius of Chiron give value about 90 km [8], [2], [9], therefore the mass of Chiron is 100 times under its mass obtained from the SBN radius and its impact upon the orbits of Jupiter, Saturn and Uranus is negligible.

It seems interesting to find out which of the minor planets from the set of 51 additional asteroids degrade the fitting in the solution 4. Asteroids with the largest masses from this list are given in Table 6. It was natural to suppose that the SBN mass of at least one of them is overestimated. So we attempted to estimate masses of these five asteroids including them into the general list of the solved-for parameters in our analysis of the ranging to martian landers and to the inner planets (solution 5). Obtained masses of the five asteroids are presented in the column 7 of Table 6 headed $m_3^{(1)}$. Estimates of these masses from solution 6 are given in the last column headed $m_3^{(2)}$. The masses obtained indeed appear to be considerably smaller than those derived from the SBN database. While the masses of (152) Atala and (190) Ismene estimated in this way are in a qualitative agreement with the SBN based value, those of (33) Polyhymnia, (64) Angelina and (675) Ludmilla are considerably smaller. The ephemeris of Mars is especially sensitive to the error of the mass of (33) Polyhymnia and (in a somewhat lesser degree) of the asteroids (675) Ludmilla, and (64) Angelina. Thus it became clear that the SBN masses of these asteroids are too large and it is the main factor that degrades the solution 4.

For comparison in the columns 3-6 of Table 6 there are given radii with the corresponding masses R_1, m_1 and R_2, m_2 of these five planets; R_1 is taken from SBN, R_2 is provided to us by Williams (private communication).

Masses of Atala and Ismene probably also need essential corrections but they cannot be estimated reliably from the lander data as these asteroids only slightly affect the orbit of Mars. However it may be noticed that our dynamical estimates do not contradict to the radii R_2 by Williams based on some astrophysical data. It is noteworthy that for Polyhamia the value R_2 is very close to the effective radar radius [27].

Table 6. Minor planets with uncertain values of masses

Planet	Type	R_1	m_1	R_2	m_2	$m_3^{(1)}$	$m_3^{(2)}$
(33) Polyhymnia	S	129.6	12.4	31.0	0.17	-2.3	2.0
						± 3.4	± 1.9
(64) Angelina	S	194.3	41.9	30.0	0.15	0.5	2.2
						± 5.3	± 4.6
(152) Atala	S	143.4	16.8	31.5	0.18	13.6	0.8
						± 11.2	± 7.5
(190) Ismene	C	201.6	23.8	79.5	1.46	43.9	-0.4
						± 27.1	± 11.5
(675) Ludmilla	S	174.0	30.1	38.0	0.31	-12.5	-1.3
						± 10.2	± 3.7

At this stage it was decided to investigate whether the results might be improved if the overall perturbing effect of other asteroids (apart of those which are directly integrated) is modelled by potential of a circular ring in the ecliptic plane. It seems plausible that there exists a large number of asteroids of the main belt which are too small to be observed from the Earth, but their summary perturbating action upon the orbit of Mars is not negligible. Thus in the list of estimated parameters the mass of the ring and its radius were included and then the described above experiments (solutions 1, 3, 5) were repeated. Statistically significant estimates have been obtained both for the mass M_{ring} (with the formal errors about $50 \cdot 10^{-12} M_{\odot}$) and for the mean radius R_{ring} of the ring (with the formal error 0.06 AU). Analytical expressions of the perturbing force from the ring are given in Appendix A. In Table 5 solutions 6–8 are those for which the parameters M_{ring} and R_{ring} also have been estimated. The solution 6 is an analogue of the solution 1 (300 asteroids with the DE405 masses), the solution 7 corresponds to the solution 3 (351 asteroids with radiometric radii), and the solution 8 corresponds to the solution 5 (another set of the additional 51 asteroids, the mentioned 5 asteroids with doubtful masses also being estimated). In the last line of Table 5 the estimated mass of the asteroid belt M_{sum} is given (being obtained as the total mass M_{tot} of the asteroids the perturbations from which are taken into account by the direct simultaneous integration and the mass of remaining asteroids modelled as the ring). Dispersion of the three solutions is probably due to errors of the individual masses adopted in the three versions of numerical integrations. From the dispersion it seems that the most plausible estimate of the mass M of the asteroid belt is

$$M = (1800 \pm 100) \cdot 10^{-12} M_\odot. \tag{1}$$

To check this estimate we can apply a theoretical distribution of the number $N(r)$ of minor planets with radii exceeding r (the distribution is based on a collisional model described in [12], [14], [31]). For the density $dN(r)$ of this distribution the following expression holds true:

$$dN(r) = -\beta r^{-3.5} dr, \tag{2}$$

where $\beta > 0$ is a constant.

Let $M(r)$ be the total mass of all asteroids with radii greater than r. In supposition that some mean density ρ may be used to calculate masses of asteroids from their volumes we obtain after integration the following expression for the distribution $M(r)$:

$$M(r) = \rho \int \frac{4}{3}\pi r^3 dN(r) = \beta_1 r^{0.5} + \beta_0,$$

with some constants β_0 and β_1. The constants β and β_1 are connected by the relation

$$\beta_1 = -\frac{8\pi}{3}\rho\beta. \tag{3}$$

Now we can evaluate the constants β_0, β_1 by fitting the distribution to the set of 300 asteroids which masses and radii may be considered as reliably estimated. It is reasonable to suppose that there are no significant effects of the observational selection in this region of changing of r where the asteroids are considerably big. After that we can extrapolate the derived distribution for $r \to 0$ and compare $M(0)$ with the estimate (1).

The expression for the value $M(r)$ obtained after the fitting is the following one (r is in kilometers, M is in $10^{-12} M_\odot$) :

$$M = 1765 - 82.9\sqrt{r}. \tag{4}$$

We see that indeed the total mass $M(0)$ of the asteroid belt is $1765 \cdot 10^{-12} M_\odot$ which value is in accordance with our finding based on the study of the perturbations in the orbit of Mars. In Fig. 4 both the curve of this distribution and the experimental data (for about 2000 minor planets) are depicted.

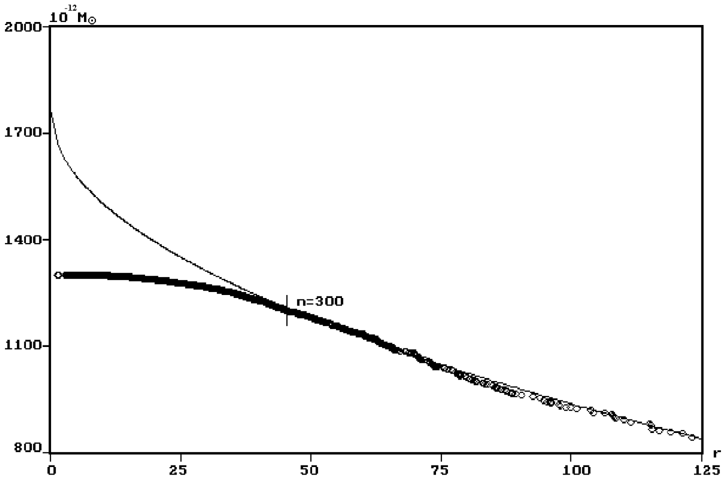

Figure 4. Distribution of masses versus radii

Let us return to the distribution (2). The parameter β may be calculated now with the help of equation (3) in which $\beta_1 = -82.9$ as it follows from (4). Here the mean density $\rho = 1.7 g/cm^3$ has to be expressed in the units $10^{-12} M_\odot/km^3$. With $M_\odot = 1.99 \cdot 10^{33} g$ we have $\rho = 0.85 \cdot 10^{-6}$. Thus we obtain

$$\beta = 11.6 \cdot 10^6.$$

With this value for β the distribution (2) shows that the expected number of the minor planets with radii of about one kilometer is $12 \cdot 10^6$ and it is about 9000 when the radii are in the range 10 km to 20 km. The typical mass of asteroids from the last subset is about $0.01 \cdot 10^{-12} M_\odot$ with the total mass $\approx 100 \cdot 10^{-12} M_\odot$ which value is not negligible if one processes the observations of the landers. Even if the mathematical problem of computation of the individual perturbations from each planet of such large set could be solved, the accumulated error of the perturbing accelerations (due to uncertainty of the individual perturbing masses) makes this direct approach unworkable. It seems that the overall perturbing effects of the asteroids of the main belt which are not in the list of 300-350 largest asteroids may be taken into account in the model of the perturbing ring and such approach has no reasonable alternative at present.

It seems useful to express the right part of the distribution (2) in terms of the absolute magnitude H; thereafter it would be possible to compare it with the all bulk of numbered asteroids. We apply the following relation between radius r (in kilometers) of an asteroid and H:

$$\lg r = 3.1 - 0.2H, \tag{5}$$

taken from [11].

Then instead of (2) we obtain:

$$dN(H) \approx 0.2 \ln 10\ \beta r^{-2.5} dH = 0.2 \ln 10\ \beta 10^{-2.5(3.1-0.2H)} dH.$$

For the unit interval of magnitudes and in the logarithmic scale this expression becomes:

$$\lg dN(H) \approx 0.5H - 1.02. \tag{6}$$

In Fig. 5 the line A presents graphically this relation in the plane $(\lg dN, H)$. The black circles are the values obtained after calculating the total number of minor planets at the intervals of the magnitudes $(H \div H{+}1)$. (The experimental data are taken from the dataset in our disposal that includes about 13000 asteroids). One can see that for $H < 8$ the slope of the theoretical curve corresponds to the experimental dependence of $\lg dN$ on H. Supposing that there is no observational selection for $H < 8$ we can calibrate the dependence given by (6) by the experimental data in this region of the magnitudes. Then instead of (6) we obtain

$$\lg dN(H) \approx 0.5H - 1.52. \tag{7}$$

The line B corresponds to this functional dependence. It is interesting that the distribution (7) can be obtained after applying a small constant correction of magnitude 0.2 to the starting dependence of $\lg r$ on H (5). Then instead of (5) we have to set:

$$\lg r = 3.3 - 0.2H.$$

Such small correction is well within the uncertainty of the relation (5) (for instance due to the adopted albedo or the photometric system).

The data presented by Fig. 5 make it possible to estimate the expected number of the minor planets in the asteroid belt which are not yet discovered in given intervals of absolute magnitudes (see Table 7). At the last column of the table the ratio of the discovered minor planets in the unit intervals of absolute magnitude to the expected number in this interval is given (in percents). One can see that the expected number of asteroids of the main belt for which the magnitude $H < 14$ is about 150000, and about 10 % of such asteroids has been already discovered.

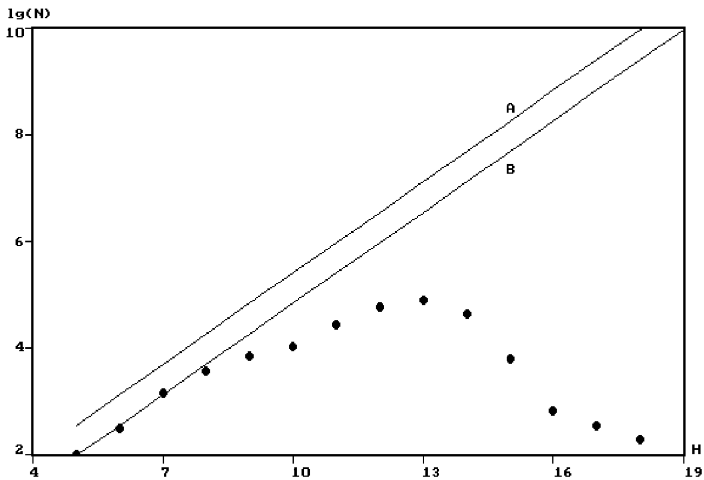

Figure 5. Distribution of lg N versus magnitudes H

Table 7. Expected (N_p) and observed (N_o) numbers of asteroids

H	N_o	N_p	%
5-6	10	10	100
6-7	27	30	100
7-8	102	95	100
8-9	233	300	80
9-10	407	950	40
10-11	606	3000	20
11-12	1375	9500	15
12-13	2666	30000	10
13-14	3447	96000	5
14-15	2056	300000	1

5. Concluding remarks

The main conclusions of this study may be summarized in the following way:

1. In order to construct planetary ephemerides which could match the existing and planning measurements of ranging to martian landers it is necessary to take into account the perturbations from all asteroids with masses greater $10^{-12}M_\odot$. It seems that the further extension of the list of 300 asteroids for which the perturbations are accounted in the DE403/DE405 cannot noticeably improve the accuracy of ephemerides of the major planets.
2. The asteroid masses derived from close encounters with other asteroids are practically useless if the resulting errors exceed $10^{-11}M_\odot$ (because the astrophysical methods provide considerably better accuracy). At present by the dynamical method such accuracy is reached only in a few cases.
3. Perturbing effects of the asteroids of the smaller masses (that do not enter the mentioned list) may be effectively described by the model of a perturbing ring. So we believe that at present the optimal way to account for all perturbations in the motion of Mars and Earth is to integrate simultaneously the equations of motion of the major planets and the 300 biggest asteroids, allowing for the perturbing accelerations from the ring as a model of the perturbations from the remaining ones.

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Appendix A

Perturbing force of the asteroid ring

Let r, v are radius vector and orbital longitude of a perturbed body, r_p, v_p are those of a perturbing asteroid. Perturbing force $\overline{F}$ acting the body is given as

$$\overline{F} = -grad\ U$$

where

$$U = gm \frac{1}{\sqrt{r^2 - 2rr_p \cos(v - v_p) + r_p^2}}.$$

Supposing that asteroids are distributed uniformly along a circular orbit in the plane of orbit of the perturbed body then the force function may be presented as the integral

$$U = gM \int_0^{2\pi} \frac{dv_p}{\sqrt{r^2 - 2rr_p \cos(v - v_p) + r_p^2}},$$

where M is the total mass of the asteroids. We are interested in the case of the perturbations on the orbits of Mars and the Earth, so $r < r_p$ and elliptical integral may be expressed in the terms of hypergeometric function F (see [4]):

$$U = \frac{gM}{r_p} F\left(0.5, 0.5, 1; \alpha^2\right),$$

and the standard notations are used:

$$F(a, b, c; z) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n, \tag{1}$$

$$(q)_n = q(q+1)...(q+n-1).$$

For the derivative needed to calculate the gradient the following expression may be applied:

$$\frac{d}{d} F(a, b, c; z) = \frac{ab}{c} F(a+1, b+1, c+1; z).$$

Then for the perturbing force of the ring we obtain

$$\overline{F} = 2\frac{gM\alpha}{r_p^2}\frac{d}{d\alpha^2}F\left(0.5, 0.5, 1; \alpha^2\right) grad\ r,$$

or

$$\overline{F} = \frac{1}{2}\frac{gM}{r_p^3}\overline{r}F\left(1.5, 1.5, 2; \alpha^2\right).$$

The simplest way for calculating the hypergeometric function at the right part of this expression is the straightforward applying the relation (1). In our case $\alpha^2 \approx 0.2$ and the series quickly converges.

Appendix B

Masses of 357 minor planets

In Table 8 the conclusive values of masses of 357 asteroids are given. The symbol ⋆ marks the asteroids for which there are no data in the SBN database. The symbol X means the unknown type of asteroids (the density $2g/cm^3$ is applied to compute the mass in this case).

Table 8. Masses of 357 minor planets in $10^{-12}M_\odot$

	planet		mass		planet		mass
1	Ceres	C	476.00	26	Proserpina	S	0.66
2	Pallas	C	108.00	27	Euterpe⋆	S	1.58
3	Juno	S	10.09	28	Bellona	S	1.41
4	Vesta	S	135.00	29	Amphitrite	S	7.38
5	Astraea	S	1.34	30	Urania	S	0.79
6	Hebe	S	4.90	31	Euphrosyne	C	5.40
7	Iris	S	5.79	32	Pomona	S	0.39
8	Flora	S	1.97	33	Polyhymnia⋆	S	0.17
9	Metis⋆	S	3.45	34	Circe	C	0.59
10	Hygiea	C	27.87	35	Leukothea	C	0.45
11	Parthenope	S	2.93	36	Atalante	C	0.46
12	Victoria	S	1.10	37	Fides	S	0.99
13	Egeria	C	3.51	38	Leda	C	0.62
14	Irene⋆	S	3.27	39	Laetitia	S	2.77
15	Eunomia	S	13.99	40	Harmonia	S	0.94
16	Psyche	M	25.33	41	Daphne	C	2.12
17	Thetis	S	0.57	42	Isis	S	0.84
18	Melpomene	S	2.23	43	Ariadne	S	0.19
19	Fortuna⋆	C	2.86	44	Nysa	S	0.27
20	Massalia	S	2.37	45	Eugenia	C	3.46
21	Lutetia	M	1.35	46	Hestia	C	0.80
22	Kalliope	M	8.96	47	Aglaja	C	0.82
23	Thalia	S	0.96	48	Doris	C	4.08
24	Themis⋆	C	2.78	49	Pales	C	1.28
25	Phocaea	S	0.33	50	Virginia	C	0.24

Table 8. Masses of 357 minor planets in $10^{-12} M_\odot$ (continuation)

	planet		mass		planet		mass
51	Nemausa	C	1.26	90	Antiope	C	0.68
52	Europa	C	10.77	91	Aegina	C	0.52
53	Kalypso	C	0.60	92	Undina	M	3.13
54	Alexandra	C	1.76	93	Minerva	C	1.09
55	Pandora	M	0.42	94	Aurora	C	3.36
56	Melete	C	0.57	95	Arethusa	C	1.07
57	Mnemosyne	S	1.10	96	Aegle	C	1.89
58	Concordia	C	0.33	97	Klotho	M	0.89
59	Elpis	C	1.85	98	Ianthe	C	0.45
61	Danae	S	0.40	99	Dike	C	0.15
62	Erato	C	0.35	100	Hekate	S	0.53
63	Ausonia	S	0.89	102	Miriam	C	0.22
64	Angelina*	S	0.15	103	Hera	S	0.58
65	Cybele	C	5.20	104	Klymene	C	0.73
68	Leto	S	1.44	105	Artemis	C	0.67
69	Hesperia	M	3.99	106	Dione	C	1.23
70	Panopaea	C	0.72	107	Camilla	C	4.77
71	Niobe	S	0.47	109	Felicitas	C	0.27
72	Feronia	C	0.25	110	Lydia	M	0.95
74	Galatea	C	0.65	111	Ate	C	0.96
75	Eurydike	M	0.27	112	Iphigenia	C	0.15
76	Freia	C	2.46	114	Kassandra	C	0.39
77	Frigga	M	0.48	115	Thyra	S	0.40
78	Diana	C	0.70	116	Sirona	S	0.30
80	Sappho	S	0.37	117	Lomia	C	1.31
81	Terpsichore	C	0.68	120	Lachesis	C	2.02
83	Beatrix	C	0.21	121	Hermione	C	3.61
84	Klio	C	0.20	122	Gerda	S	0.45
85	Io	C	1.36	124	Alkeste	S	0.35
86	Semele	C	0.72	127	Johanna*	C	0.65
87	Sylvia	C	7.05	128	Nemesis	C	2.61
88	Thisbe	C	4.47	129	Antigone	M	2.72
89	Julia	S	2.77	130	Elektra	C	2.38

Table 8. Masses of 357 minor planets in $10^{-12} M_\odot$ (continuation)

	planet		mass		planet		mass
134	Sophrosyne	C	0.63	190	Ismene*	C	1.46
135	Hertha	M	0.74	191	Kolga	C	0.41
137	Meliboea	C	1.21	192	Nausikaa	S	0.86
139	Juewa	C	1.52	194	Prokne	C	1.89
140	Siwa	C	0.53	195	Eurykleia	C	0.25
141	Lumen	C	0.88	196	Philomela	S	2.14
143	Adria	C	0.28	200	Dynamene	C	0.82
144	Vibilia	C	1.11	201	Penelope	M	0.48
145	Adeona	C	1.31	202	Chryseis	S	0.43
146	Lucina	C	0.90	203	Pompeja	C	0.62
147	Protogeneia	C	0.92	205	Martha	C	0.20
148	Gallia	S	0.79	206	Hersilia*	C	0.52
150	Nuwa	C	1.39	209	Dido	C	1.16
152	Atala*	S	0.18	210	Isabella	C	0.25
153	Hilda	C	1.92	211	Isolda	C	1.14
154	Bertha	C	2.49	212	Medea	C	0.96
156	Xanthippe	C	0.72	213	Lilaea	C	0.21
159	Aemilia	C	0.80	216	Kleopatra	M	3.74
160	Una	C	0.21	221	Eos	S	0.91
162	Laurentia	C	0.41	224	Oceana	M	0.48
163	Erigone	C	0.16	225	Henrietta	C	0.68
164	Eva	C	0.46	227	Philosophia	X	0.37
165	Loreley	C	1.47	229	Adelinda	C	0.31
168	Sibylla	C	1.28	230	Athamantis	S	0.99
171	Ophelia	C	0.63	231	Vindobona	S	0.42
173	Ino	C	1.44	233	Asterope	C	0.44
175	Andromache	C	0.44	236	Honoria	S	0.51
176	Iduna	C	0.68	238	Hypatia	C	1.33
179	Klytaemnestra	S	0.36	240	Vanadis	C	0.44
181	Eucharis	S	0.86	241	Germania	C	1.70
184	Dejopeja	M	0.44	245	Vera	S	0.42
185	Eunike	C	1.58	247	Eukrate	C	0.92
187	Lamberta	C	0.88	250	Bettina	M	0.86

Table 8. Masses of 357 minor planets in $10^{-12} M_\odot$ (continuation)

	planet		mass		planet		mass
257	Silesia	S	0.27	356	Liguria	C	0.86
259	Aletheia	C	2.23	357	Ninina	C	0.48
260	Huberta	C	0.37	358	Apollonia	C	0.27
266	Aline	C	0.52	360	Carlova	C	0.63
268	Adorea	C	1.03	361	Bononia	C	1.16
275	Sapientia	C	0.54	362	Havnia*	C	0.34
276	Adelheid	C	0.73	363	Padua*	C	0.33
283	Emma	C	1.21	365	Corduba	C	0.48
286	Iclea	C	0.32	366	Vincentina	C	0.33
287	Nephthys	S	0.24	369	Aeria	M	0.33
303	Josephina	C	0.38	372	Palma	C	2.61
304	Olga	C	0.11	373	Melusina	C	0.35
308	Polyxo	C	1.14	375	Ursula*	C	3.61
313	Chaldaea	C	0.36	377	Campania	C	0.30
322	Phaeo	M	0.54	379	Huenna	C	0.31
324	Bamberga	C	5.01	381	Myrrha	C	0.68
325	Heidelberga	M	0.64	382	Dodona	M	0.30
326	Tamara	C	0.35	385	Ilmatar	S	0.57
328	Gudrun	S	1.21	386	Siegena	C	1.85
329	Svea	C	0.18	387	Aquitania	S	0.84
334	Chicago	C	1.73	388	Charybdis	C	0.62
335	Roberta	C	0.29	389	Industria	S	0.37
336	Lacadiera	C	0.13	393	Lampetia	C	0.43
337	Devosa	M	0.35	401	Ottilia	X	0.58
338	Budrosa	M	0.33	404	Arsinoe	C	0.37
344	Desiderata	C	0.94	405	Thia	C	0.77
345	Tercidina	C	0.35	407	Arachne	C	0.33
346	Hermentaria	S	0.91	409	Aspasia	C	1.67
347	Pariana	M	0.22	410	Chloris	C	0.73
348	May	X	0.36	412	Elisabetha	C	0.29
349	Dembowska	S	2.06	416	Vaticana	S	0.50
350	Ornamenta	C	0.67	419	Aurelia	C	0.82
354	Eleonora	S	2.99	420	Bertholda	C	1.11

Table 8. Masses of 357 minor planets in $10^{-12} M_\odot$ (continuation)

	planet		mass		planet		mass
423	Diotima	C	3.61	532	Herculina	S	8.55
424	Gratia	C	0.26	535	Montague	C	0.16
426	Hippo	C	0.84	536	Merapi	C	1.39
431	Nephele	C	0.33	545	Messalina	C	0.54
433	Eros	S	0.003	554	Peraga	C	0.34
441	Bathilde	M	0.54	558	Carmen	M	0.32
442	Eichsfeldia	C	0.11	566	Stereoskopia	C	1.89
444	Gyptis	C	1.73	567	Eleutheria	C	0.32
449	Hamburga	C	0.24	568	Cheruskia	C	0.25
451	Patientia	C	4.30	570	Kythera	S	0.84
454	Mathesis	C	0.21	576	Emanuela	X	0.33
455	Bruchsalia	C	0.24	579	Sidonia	S	0.50
466	Tisiphone	C	0.62	595	Polyxena	C	0.52
469	Argentina	C	0.75	596	Scheila	C	0.56
471	Papagena	S	1.89	601	Nerthus	S	0.30
476	Hedwig	C	0.63	602	Marianna	C	0.77
478	Tergeste	S	0.37	618	Elfriede	C	0.68
481	Emita	C	0.56	626	Notburga	C	0.40
488	Kreusa	C	1.39	635	Vundtia	C	0.35
489	Comacina	C	1.07	639	Latona	S	0.28
490	Veritas	C	0.63	654	Zelinda	C	0.82
491	Carina	C	0.36	663	Gerlinde	C	0.40
498	Tokio	C	0.21	674	Rachele	S	0.72
505	Cava*	C	0.54	675	Ludmilla*	S	0.31
506	Marion	C	0.46	683	Lanzia	C	0.56
508	Princetonia	C	1.14	690	Wratislavia	C	0.96
511	Davida	C	13.58	691	Lehigh	C	0.28
514	Armida	C	0.48	694	Ekard	C	0.28
516	Amherstia	M	0.59	702	Alauda	C	2.91
517	Edith	S	0.60	704	Interamnia	C	12.98
521	Brixia	C	0.62	705	Erminia	C	0.94
522	Helga	C	0.50	709	Fringilla	C	0.35
528	Rezia	X	0.33	712	Boliviana	C	0.82

Table 8. Masses of 357 minor planets in $10^{-12} M_{\odot}$ (continuation)

	planet		mass		planet		mass
713	Luscinia	C	0.46	849	Ara	M	1.31
733	Mocia	C	0.27	866	Fatme	X	0.40
739	Mandeville	C	0.48	886	Washingtonia	X	0.42
740	Cantabia	C	0.30	895	Helio	C	1.11
747	Winchester	C	1.99	909	Ulla	C	0.62
748	Simeisa	C	0.43	912	Maritima	X	0.33
751	Faina	C	0.54	914	Palisana	C	0.17
758	Mancunia	S	0.45	980	Anacostia	S	0.48
760	Massinga	S	0.28	1015	Christa	C	0.37
762	Pulcova	C	1.03	1021	Flammario	C	0.39
769	Tatjana	C	0.38	1036	Ganymed	S	0.05
772	Tanete	C	0.67	1093	Freda	C	0.62
773	Irmintraud	C	0.34	1177	Gonnessia	C	0.31
776	Berbericia	C	1.99	1268	Libya	C	0.33
780	Armenia	C	0.32	1269	Rollandia	C	0.46
784	Pickeringia	X	0.37	1303	Luthera	X	0.36
786	Bredichina	C	0.29	1345	Potomac	S	0.35
788	Hohensteina	C	0.45	1390	Abastumani	C	0.40
790	Pretoria	C	1.92	1467	Mashona	C	0.50
791	Ani	C	0.43	1902	Shaposhnikov	C	0.37
804	Hispania	C	1.47	2060	Chiron	C	2.09
814	Tauris	C	0.54				

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