Dynamical models of the Solar system and accuracy of planetary ephemerides

E. V. Pitjeva(1), D. A. Pavlov(1), N. P. Pitjev(2)

(1) Institute of Applied Astronomy of RAS, St. Petersburg, Russia, (2) St. Petersburg State University, St. Petersburg, Russia (evp@ipa.nw.ru /Fax +7-812-2751119)

Dynamical models

Dynamical models of the solar system are models of motion of planets, including also, for accuracy, the Sun, the Moon, asteroids and Trance-Neptunian Objects. Those models are used for creation of planetary ephemerides. Until the 1960s, ephemerides of individual planets were constructed by analytical methods by Le Verrier, Hill, Newcomb, and Clemens. Experiments performed in deep space and the introduction of new observational techniques (radar ranging, lunar laser ranging, VLBI measurements, etc.) required the development of precise planetary ephemerides. The errors of the best ranging observations do not exceed several meters, making it necessary to compute delay times correctly up to the 12th decimal digit. Such a high precision requires the construction of an appropriate model of the motion of celestial bodies. This is a serious problem; the easiest way to solve it is to perform computer-assisted numerical integration of the equations of motion of the planets and the Moon.

In the 1960-1970s, several research groups in the United States and Russia developed numerical theories to support space flights. The EPM ephemerides (Ephemerides of Planets and the Moon) of the IAA RAS have been in production since 1974, about 10 years later than the well-known DE (Development Ephemeris of JPL – Jet Propulsion Laboratory, USA) ephemerides. In 2000s French Integrateur Numerique Planetaire de l'Observatoire de Paris – INPOP ephemerides was connected to these ephemerides. All three modern main planetary ephemerides — DE, EPM, and more recent INPOP are based on General Relativity involving the relativistic equations of celestial bodies motion and light propagation as well as the relativistic time scales. Common to these planetary ephemerides is simultaneous numerical integration of the equations of motion of the nine major planets, the Sun, the Moon, and the lunar physical libration performed in the Parameterized Post-Newtonian n-body metric for General Relativity in the TDB (Time Dynamical Barycentric) time scale. Thus the dynamical models of these ephemerides include the Newtonian and relativistic mutual perturbations of all planets, the Sun and the Moon.

The motion of the barycenter of the Earth-Moon system is appreciably perturbed by the Moon itself. The resonant behavior of the coupling between orbital and rotational motions of the Moon makes it necessary that the theories of motion of the Earth and the Moon should exist in a common dynamical model.

The solar oblateness causes secular variations in the orbital elements of planets, with the exception of semi-major axes and eccentricities. Hence, starting with DE405(1998) and EPM2000(2001), ephemerides are integrated considering a nonzero value of the quadrupole moment of the Sun. That value is currently determined during the processing of high-precision ranging measurements.

The largest asteroids are massive enough to significantly affect the orbits of other bodies in the solar system. Even the earliest DE and EPM ephemerides took into account perturbations from several largest asteroids: Ceres, Pallas, Vesta, Iris, and Bamberga. The experiments undertaken to process Viking landers data showed that it was impossible to achieve a good representation of these observations taking into account the perturbations caused by only 3-5 largest asteroids. The perturbations from 300 asteroids were taken into account starting with DE230 and were implemented in the well-known DE403 (1995) ephemerides. The perturbations from 300 asteroids were taken into account since EPM98 (1998).

The perturbations from the remaining small asteroids were modeled by additional perturbations from a massive ring in the plane of the ecliptic with a uniform mass distribution. That approach was proposed by Krasinsky et al. (2002) and implemented in EPM2004 ephemerides. However, the asteroid belt is a region of the solar system placed roughly between the planets Mars and Jupiter, where the greatest concentration of asteroid orbits (approximately 93.4% of all numbered minor planets) can be found. The area of the asteroid belt laying between the radial distances 2.06 and 3.27 au those can be considered as the inner and outer boundaries of the main part of the main belt. In the first time, in the EPM2013 ephemerides, a massive two-dimensional asteroid annulus has been used for modeling the total perturbation from the remaining small asteroids, instead of a one-dimensional asteroid ring of our previous ephemerides. Finding the gravitational potential and its derivatives for the flat two-dimensional homogeneous annulus leads to expressions involving complete elliptic integrals of the 1st, 2nd and 3rd kind. Calculations can be simplified by applying the Landen transformation, and finding the values of hypergeometric functions.

Hundreds of Trans-Neptunean Objects (TNO) that were discovered lately also could affect the motion of planets, especially the outer ones. Since EPM2008 the dynamical model of the EPM ephemerides includes Eris (a dwarf planet discovered in 2003, more massive than Pluto) and 20 of the largest TNOs into simultaneous integration, then since EPM2011 perturbations from the one-dimensional ring of TNO with the radius of 43 au were added in the dynamical model of EPM2011 ephemerides. Since EPM2013 we take into account the 30 individual TNO.

Thus the brief evolution of dynamical models of the planet part of EPM ephemerides is shown:

EPM87 (Krasinsky et al., 1993):

- Mutual perturbations from the major planets, the Sun, the Moon and 5 more massive asteroids;
- the relativistic perturbations.

EPM98 (1998):

- Perturbations from the other 296 asteroids chosen due to their strong perturbations upon Mars and the Earth.

EPM2000 (2001):

- Perturbations due to the solar oblateness J_2, that is currently determined during the processing of high-precision ranging measurements.

EPM2004 (2005):

- Perturbation from the massive one-dimensional asteroid ring with the constant mass distribution. The two parameters characterizing the ring (its mass M_r and radius

R_r) were included into the set of improved parameters. That approach was proposed by Krasinsky et al. (2001) and implemented in EPM2004 ephemerides.

EPM2008 (2010)

- Perturbations from the 21 largest Trans-Neptunian Objects (TNO).

EPM2011 (2013):

- Perturbation from a massive ring of TNO in the ecliptic plane with the radius of 43 au.

EPM2013/EPM2014 (2014):

- Perturbation from the massive two-dimensional asteroid annulus ($R_1 = 2.06$ au, $R_2 = 3.27$ au) with the estimated mass.
- perturbations from the 30 largest Trans-Neptunian Objects (TNO).

As for as the lunar part of the EPM, two dynamical models of the lunar motion are being developed at present. The first model of the lunar motion was constructed by Krasinsky and has been developed by Vasilyev and Yagudina, while the second one is the model proposed by Williams for DE430 (2013) lunar theory and is currently being implemented by Pavlov for EPM. Both models take into account a number of physical effects such as elastic tide, gravitational potential of the Moon and the Earth, friction between lunar mantle and molten core, and dissipation of energy. The parameters of the lunar part of ephemerides have been improved using about 18000 LLR observations (1970–2014).

As for the analytical models of planetary motion, their development goes on. The most accurate analytical theories of motion are represented by the VSOP series of French ephemerides developed at the IMCCE of Paris Observatory. Recently, considerable progress has been achieved for the new ephemerides, VSOP2013 and TOP2013 where the perturbations from the Moon and Pluto, 300 asteroids, solar oblateness, and relativistic effects have been taken account. However the constants of these theories have been were obtained by fitting to the numeral INPOP10a ephemerides, not to observations.

Construction of ephemerides and their accuracy

The basic workflow of the ephemeris creation and parameter determination process is that of a least-squares iteration which can be reduced to the following:

- Numerical integration of the equations of motion for the major planets, Moon, and Sun and variational equations for producing the partial derivatives.
- Computing the model observations “C” (e.g. time delays) from the produced ephemerides for the time of each observation “O”, calculating the residuals (O-C) and the partial derivatives.
- Obtaining the values of the parameters being determined, and deriving the residuals of the observations after the improvement by the least-squares adjustment; for that, observations are weighted in accordance with their a priori accuracy.

Fundamental ephemeris of planets and the Moon (EPM) have been created at IAA RAS since the 70-th of the last century. Some of their major versions EPM2004, EPM2008, EPM2011/m are available at ftp://quasar.ipa.nw.ru/incoming/EPM/. EPM2014 version has been created with the updated and refined software package ERA-8.

The accuracy of the planetary ephemerides depends on several factors: the adequacy of dynamic model of planet motion to their real motion, the quantity and quality of observational data, as well as the reduction of the observations.

The reduction of observations includes all the relevant corrections. The main reductions for the optical observations of planets are: correction for the additional phase effect (the main phase corrections were made by observers themselves) and corrections for referencing observations obtained in different catalogues to the ICRF reference frame. The reduction for ranging observations includes the relativistic corrections — the time delay (the Shapiro effect) and path-bending of the propagation of radio-signals in the gravitational fields of the Sun, Jupiter, and Saturn, and the reduction of observations from the coordinate time of the ephemerides to the proper time of the observer; the delay from the solar corona; the delay from the Earth troposphere; and the correction of planetary radar observations for topography. Although reductions of the observations are well known, more precise data need more accurate calculation. For example, processing the Cassini spacecraft data required to refine the calculation of proper time.

The number of observations has increased by an order since EPM2000 (2001) and amounts for more than 800,000 data of different types, most of the new observations are precision radio measurements. Observations are classical and modern optical observations of the outer planets and their satellites (since 1913), ranging to planets, the martian landers and spacecraft, including the data of Mariner-9, Viking, Pathfinder, MGS, Odyssey, MRO, Venus Express and Mars Express (1961-2014), VLBI spacecraft observations (1990-2014). VLBI spacecraft data around planets in the background quasars enable orientation of the planetary ephemeris in the international system ICRF with an accuracy of fractions of mas.

Several hundred of parameters are determined simultaneously while improving the planetary part of the ephemerides. In addition to the orbital elements of all the planets and the main satellites of the outer planets, this set includes masses of celestial bodies, parameters of surface topography of planets and rotation of Mars, the coefficients of the solar corona and the solar oblateness, parameters of the orientation of planet ephemerides to the ICRF, etc.

The dynamical models of the EPM ephemerides have been improved significantly. Number of observations used for improvement of ephemerides and their accuracy have increased greatly, as well as the reduction of observations.

Improvement of all these factors helped to increase the accuracy of planetary ephemerides, which was manifested in a decrease of the formal standard deviations of orbital elements of planets by several times, and for planets provided by high-precision data from spacecraft (Venus, Mars, Saturn) — by one order. The accuracy of the mass of the two-dimensional asteroid annulus has increased by 6 times that has resulted in the significant improvement the accuracy the total mass of the main asteroid belt composed from the 301 asteroids and the mass of the two-dimensional modeled ring of remaining small asteroids.

Detailed description of EPM ephemerides, their models, parameters, the accuracy and the all references are given the the paper Pitjeva E.V., Pitjev N.P., 2014. Development of planetary ephemerides EPM and their applications. - Celest. Mech. & Dyn.Astr., 119, 237-256.