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The Lyapunov spectra in spin–orbit dynamics

V. V. Kouprianov, I. I. Shevchenko

Pulkovo Observatory, St. Petersburg, Russia

We study the problem of the chaotic rotation of a minor planetary satellite around its mass center, in the planar and spatial settings. In the planar setting a satellite rotates around the axis of its maximum moment of inertia which is orthogonal to the orbit plane, while in the spatial setting it may rotate in any direction. Whether the motion is chaotic or regular, one can infer by means of calculation of the Lyapunov characteristic exponents (LCEs) [1, 2]. The LCEs describe rates of exponential divergence of close trajectories in phase space.

For a set of eleven satellites of Mars, Jupiter, Saturn, and Neptune (the satellites with known values of inertial and orbital parameters) the full Lyapunov spectra of chaotic rotation are computed numerically. Among these satellites, currently only Hyperion (S7) is confirmed to rotate chaotically. The present rotation modes of five other satellites, namely Helene (S12), Atlas (S15), Prometheus (S16), Pandora (S17), and Proteus (N8), are unknown. The rest of the satellites in our set rotate regularly in synchronous resonance; but, in any case, each of them had passed through the stage of chaotic rotation. Calculation of the LCE spectra for these satellites extends our knowledge of their dynamical history.

A satellite is modelled as a tri–axial rigid body moving in a fixed elliptic orbit. The dynamic system is described by the Euler equations [3]. The parameters of the problem are the orbital eccentricity of a satellite e and the ratios of its principal central moments of inertia A/C and B/C ($A < B < C$). These ratios characterize the dynamical asymmetry of a satellite. The LCE spectrum is computed by the HQR method in the version of von Bremen et al. [4]. This method is based on the QR decomposition of the tangent map matrix using the Householder transformations.

The maximum LCEs for the satellites in our set are estimated analytically using the separatrix map theory in the model of nonlinear resonance (here synchronous spin–orbit resonance) as a perturbed nonlinear pendulum [5, 6]. This approach is based on the hypothesis by Chirikov [1] that the dynamical entropy of the separatrix map is constant in the high–frequency limit of perturbation.

A key role in the method belongs to the average dependence of the dynamical entropy of the separatrix map upon λ (the ratio of the perturbation frequency to the frequency of small oscillations on the resonance) in the whole range of the values of λ $(0 < \lambda < +\infty)$.

Comparison of the results of numerical and analytical estimations of the maximum LCEs shows a good agreement in the case of planar rotation. It is shown also that the theory developed for the planar case is most probably still applicable in the case of spatial rotation, if the dynamical asymmetry of the satellite is sufficiently small, $A/C \gtrsim 0.8$, or/and the orbital eccentricity is relatively large, $e \gtrsim 0.02$ (but not too large, in order for the dynamical model to be valid). Otherwise the theory should be different.

It is plausible that chaotic dynamics of strongly asymmetric satellites in nearly circular orbits, in the case of spatial rotation, is determined mainly by interaction of internal coupling resonances. In order to check this, we have recomputed the LCE spectra for our set of satellites, formally assigning the orbital eccentricity to zero. Comparison with the LCEs in case of the actual eccentricities provides a hint for the development of a future theory.

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