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Relativistic celestial mechanics–2002: results and prospects

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1. Introduction

The aim of this paper is to review the present status of relativistic celestial mechanics in its broadest sense. Nowadays many physicists and astrophysicists do researches in the field of relativistic celestial mechanics. Due to these researches the subject field of celestial mechanics has been significantly extended beyond the scope of motion of the solar system bodies (weak gravitational fields). The main attention is focused now on investigating motion of astrophysical and cosmological objects such as black holes, gravitational waves, neutron stars, inspiralling compact binaries, etc. (strong gravitational fields). This review covers four main aspects of relativistic celestial mechanics, i.e. GRT and observations, GRT and astrophysics, GRT and ephemeris astronomy, and GRT and classical celestial mechanics problems.

2. GRT and observations

The detailed analysis of the current status of comparison of general relativity (weak field and strong field effects) with observations is given in very compact and elucidative form by Schäfer (2000). In spite of the present agreement between experiment and theory new theoretical developments in astrophysics and cosmology provide new motivations for pursuing the experimental tests of GRT. From this point of view the confrontation between GRT and observations is analysed by Damour (see relevant papers in http://arXiv.org, gr-qc series). restrict here by exposing the main tests of GRT as described in (Schäfer, 2000 with references therein).

GRT is based on the Equivalence Principle. A distinction is made between the Weak Equivalence Principle (WEP), Einstein Equivalence Principle (EEP) and Strong Equivalence Principle (SEP). WEP, i.e. the identity of inertial and gravitational mass, is checked now experimentally within a precision of 1×10^{-12}.

EEP, i.e. the local equivalence of gravitational and inertial fields involving the universality of gravitational redshift, is verified with a precision of 2×10^{-4}. SEP, i.e. the extension of EEP for self–gravitating test systems, is characterized by Nordtvedt parameter $\eta^G = 4\beta - \gamma - 3$, β and γ being the main PPN (Parametrized Post–Newtonian) parameters equal to 1 in GRT. The present SEP tests result in $\eta^G = -0.0007 \pm 0.0010$. On the other hand, the LLR and VLBI deflection measurements give $\gamma = 0.9996 \pm 0.0017$, $\beta = 0.9997 \pm 0.0005$.

The LLR measurements enable also to check the effect of the geodetic precession. The presently highest relative precision amounts to 5×10^{-3}. Radar measurements to planets and satellites result in an upper bound of the possible variation of the Newtonian gravitational constant of $|\dot{G}/G| < 0.6 \times 10^{-11} \mathrm{yr}^{-1}$. Quite recently, the precession of the orbital planes of the Earth's artificial satellites caused by the Earth's rotation (Lense–Thirring precession) was verified with a precision about 20%.

All these effects are characteristic for the weak gravitational fields. The most important GRT test for strong gravitational fields is related with binary pulsar motion. Two close binary radio pulsars with neutron–star companions are used now for testing strong–field effects of GRT: PSR B1913+16 and PSR B1534+12. The consistent solution for masses and orbital elements of PSR B1913+16 proves the correctness of GRT effects including the existence of gravitational waves. The precision of this test amounts presently 0.35%. In the nearest future the analysis of observations of PSR B1534+12 may be even more important permitting to measure the corresponding geodetic precession and some strong–field effects of alternative gravitation theories. The planned future space missions and ground observatories designed for direct investigation of gravitational waves will result in further tests for black holes and the big band.

3. GRT and astrophysics

Practically until two last decades of the XXth century all problems of relativistic celestial mechanics have been treated in the first post–Newtonian approximation (1PNA), i.e. within c^{-2} accuracy with respect to the Newtonian terms. In this respect relativistic celestial mechanics of that time was simpler mathematically than high–accuracy Newtonian celestial mechanics with its subsequent approximations far beyond the first order. The situation changed with the discovery of binary pulsar PSR B1913+16. To study its motion in taking into account gravitational radiation it is necessary to derive and to solve the equations of motion in 2.5PNA, i.e. within c^{-5} accuracy. It turns out that in 2PNA (c^{-4} accuracy) the N body problem does not differ qualitively from the corresponding Newtonian problem (conservative dynamical system). The qualitative difference reveals in 2.5PNA due to the loss of the energy of the system by gravitational radiation. The system becomes non–conservative and irreversible (in time). The consistent solution for the binary pulsar problem in 2.5PNA obtained by celestial mechanics techniques proved implicitly the existence of gravitational waves. To get insight into further evolution of the binary when the distance between bodies becomes less than the radius of the innermost stable circular orbit one needs to proceed further approximations with respect to the GRT small parameters. Present investigations (Jaranowski and Schäfer, 1997, 1998; Damour et al., 2000a,b, 2001) deal with 3PNA and even 3.5PNA (c^{-6} and c^{-7} accuracy, respectively). Such higher–order approximations are necessary for understanding the processes of coalescing inspiralling galaxies and gravitational wave emission. The analytical techniques applied in these investigations extend the arsenal of existing methods of celestial mechanics (as an effective one–body approach to two–body dynamics developed by Buonanno and Damour, 1999).

On the other hand, these techniques are complemented by numerical relativity methods for the regions deeply inside the innermost stable circular orbit.

4. GRT and ephemeris astronomy

First GRT–based IAU resolutions on reference systems and time scales were adopted by the IAU in 1991. This event may be regarded as recognition of relativistic character of modern ephemeris astronomy both with respect to its theoretical accuracy and observational precision. The latest IAU resolutions were adopted in 2000 (IAU, 2001). Much remains to be done for realization of these resolutions (Brumberg and Groten, 2001). IAU resolutions demand to consider two principal astronomical reference systems ICRS and ITRS as relativistic four–dimensional systems with TCB and TCG, respectively, as their time scales. In practice, these systems are often used as three–dimensional Newtonian systems in combination to TDB and TT, respectively. This fact causes a lot of confusion.

IAU(2000) resolutions involve one more system, GCRS, to be served as an intermediary between ICRS and ITRS. To avoid any GRT ambiguities in interpreting ephemeris astronomy concepts one needs even more reference systems at the barycentric and geocentric level.

Accurate analytical expression for the difference TDB–TT (in the geocentre) has been given in (Fairhead and Bretagnon, 1990 ; see also Irwin and Fukushima, 1999). Guinot (2000) pointed out the necessity to take into account the constant value of the mixed and trigonometric terms in this difference for the rigorous fulfillment of the IAU resolutions (both the time scales difference and the geodetic rotation vector are determined by differential equations and one should specify initial conditions in solving these equations).

New advances in solving the equations of light propagation are made in (Kopeikin et al.,1999) and (Blanchet et al., 2001).

The most accurate algorithms of relativistic reduction of astronomical observations are proposed by Klioner (2001) for space astrometry, by Kopeikin and

Ozernoy (1999) for binary star observations and by Klioner (1991) for VLBI observations.

Numerical planetary theories have been analysed in (Pitjeva, 2001 and references therein).

5. GRT and classical celestial mechanics problems

GRT planetary equations used now in ephemeris astronomy represent the well–known EIH (Einstein–Infeld–Hoffman) equations for the point masses. Various generalizations of these equations for the rotating extended masses (including the equations for rotational motion) were proposed in the second half of the last century but the physical structure of the bodies was often considered there rather formally (not violating mathematical correctness of these equations). Only recently the physically reliable equations for the binaries with consideration of spin and quadrupole moments were derived in (Xu et al., 1997). Various ways to derive the explicit rotational equations of motion of celestial bodies are discussed in (Klioner and Soffel, 1998, 1999).

Earth's satellite equations were analysed in (Damour and Esposito–Farèse, 1994) with respect to the major GRT effects. Klioner (2001) gave these equations in explicit form with accuracy more than enough for present practical purposes.

All these equations intended to study the motion of the solar system bodies are derived in 1PNA. Using the 2.5PNA equations for binary pulsar it is possible to study the motion of a test particle in the binary pulsar gravitational field (Brumberg, 2002). As a simple example of a non–conservative and irreversible (in time) type of motion one may consider the relativistic restricted quasi–circular three–body problem with gravitational radiation taken into account. In the simplest approximation the equations of motion of such problem have formally the Newtonian form with coordinates of the binary ($K = 1, 2,\ i = 1, 2$)

$$x^i_K = -(-1)^K \frac{M_K}{M} R^i, \ \ R^i = R \begin{pmatrix} \cos u \\ \sin u \end{pmatrix}, \ \ R = A(1 - 2k\Lambda), \ \ u = \Lambda + \tfrac{3}{2} k\Lambda^2,$$

with

$$\Lambda = Nt + \Lambda_0, \quad k = \tfrac{32}{5} c^{-5} N^5 A^5 \mu, \quad \mu = \frac{M_1 M_2}{M^2},$$

N, A, M being mean motion, semi–major axis and sum of mass of the binary, respectively. The simplest quasi–circular plane solution for a test particle ar large distance from the binary reads

$$\begin{pmatrix} x \\ y \end{pmatrix} = a\Big(1 + 3k\mu \frac{A^2}{a^2}\Lambda\Big) \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}, \quad \varphi = \lambda - 3k\mu \frac{A^2}{a^2}\frac{n}{N}\Lambda^2, \quad \lambda = nt + \varepsilon,$$

n, a being unperturbed values for the mean motion and semi–major axis of a test particle.

6. Conclusion

It is of interest to see the list of the most important unsolved problems in astrophysics given by Wesson (2001). The list contains 20 fundamental problems of modern astrophysics. It seems that at least a quarter of them should be treated by methods of relativistic celestial mechanics underlying its role in setting closer relationship between astrophysics and celestial mechanics.

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