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Analytical theories of the motion of the planets and of the rotation of the rigid Earth

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1. Planetary theories

The improvement of the techniques of observation and the needs for the spacecraft navigation make necessary the construction of high precision solutions for the motion of the planets. From VSOP82 and VSOP87 solutions (Bretagnon, 1982, Bretagnon and Francou,1988), we undertook the construction of analytical theories of the motion of planets at the 10^{-10} (0.02 mas) level. If the quality of the observations of the giant planets does not require such a precision, such a level is necessary for Mercure, Venus, the Earth and Mars and is reached for the first three planets. It is not the case for Mars because of the uncertainty of the masses of a great number of disturbing asteroids. The intrinsic quality of the resolution of the equations is such as the error on the position of Mars for fixed values of the mass of the asteroids is about 500 meters (20×10^{-10} radian) over one century and of 20 meters (10^{-10} radian) over 10 years. But, as showed by Standish and Fienga (2002), the masses of the asteroids do not make it possible to envisage the position of Mars to better than 1 kilometer over 10 years.

The constants of integration are in the course of determination (Fienga, 1999) by comparison with the observations.

The planetary solutions are expressed as function of TCB (Barycentric Coordinate Time) (Brumberg, 1991) and are used to calculate the relations between the time scales TCB and TCG (Geocentric Coordinate Time).

$$TCG = TCB\,(1 - L_C) + \text{periodic terms}.$$

The Table 1 gives a comparison de L_C with the determination by Fukushima (1995) and by Irwin and Fukushima (1999). In this latter solution the uncertainty is 2×10^{-17}.

Table 1. Secular term L_C of TCB$-$TCG.

Solution	$L_C \times 10^8$
Fukushima (1995)	1.480 826 845 7
Irwin and Fukushima (1999)	1.480 826 867 41
Our result	1.480 826 866 70

2. Reference systems

Until now, the analytical solutions were built in the ecliptic and the equinox J2000. But, such a reference frame depends on the solution considered, analytical or numerical.

Thus, in the current state of the development of the analytical solutions of the motion of planets, the mean plane of the ecliptic is obtained from the ICRF by two rotations:

1) a rotation about the z axis of φ with

$$\varphi = -\,0.053\,727'' = -\,0.000\,000\,260\,476 \text{ radian}$$

2) a rotation about the x axis of ε with

$$\varepsilon = 23°26'21.408\,800'' = 0.409\,092\,614\,174 \text{ radian.}$$

Following the adoption of the ICRF by IAU in 1997, we propose to define a reference frame close to the ecliptic, the "ecliptic ICRF", obtained from the ICRF by a rotation about the x axis of $\varepsilon_{\mathrm{ICRF}}$ where

$$\varepsilon_{\mathrm{ICRF}} = 0.409\,092\,614 \text{ radian exactly.}$$

It is in this reference frame that will be built henceforth the analytical planetary solutions.

3. Earth rotation

The analytical solutions VSOP of the motion of planets as well as the solution ELP2000/82 of the Moon (Chapront-Touzé and Chapront, 1983) are used to build the solution SMART97 of the rotation of the rigid Earth (Bretagnon *et al*, 1998). We determine the three angles of Euler ψ, ω and φ with the accuracy of 2 μas for ψ and φ and of 0.6 μas for ω on the interval 1970–2020.

It is not desirable to separate precession and nutation today, also the solutions are built globally for the polynomial parts and the periodic and of Poisson parts by taking into account simultaneously the perturbations by the Moon, the Sun and all the planets.

Let us note that variables such as θ, z and ζ are singular (they are not defined in J2000) and do not take place any more to be considered.

The MHB2000 model (Mathews *et al*, 2002), developed for nutations with period higher than two days, was adopted by IAU in 2000 and was applied to the solution of Souchay *et al* (1999). In 2002, IERS recommended to build the polynomials of precession which do not appear in the solution of Souchay *et al* in order to imprve the solution of Lieske *et al* (1977). From the solution of the rotation of the rigid Earth SMART97 more precise and which contains the polynomials of precession, we build a solution of the rotation of the nonrigid Earth using the MHB2000 model as well as the diurnal and subdiurnal part starting from a model under development by Mathews (2002).

Lastly, starting from the dynamical quantities ψ, ω, φ and of the geodetic precession-nutation of Brumberg (1997), we calculate the kinematical quantities ψ_K, ω_K, φ_K.

The current solutions of the rotation of the Earth are reckoned from the ecliptic and the equinox J2000. They will have to be calculated again using better solutions of the disturbing bodies and by referring them to the 'ecliptic ICRF'.

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