IAA Transactions, No. 8, "Celestial Mechanics", 2002

On the development of M. L.Lidov's techniques on the evolution of satellite orbits

M. A. Vashkov'yak

Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences, Moscow, Russia

In this paper basic results of the investigations carried out by M. L. Lidov on the evolution of satellite orbits are presented. They have received further theoretical and applied development.

1. As early as in 1961 Lidov initiated new approach in celestial mechanics aimed to investigate the orbital evolution of artificial and natural satellites of the planets. His methods allowed to calculate sufficiently exactly the orbital parameters of artificial Earth satellites and to get qualitative conclusions about the behavior of satellite orbits of different classes on the basis of simplified integrable problem [4,5], i.e. the so called double–averaged Hill's problem. This problem describes the evolution of the satellite orbit under the effect of secular perturbations of the distant attracting point in the first approximation. Two integrals of this problem c_0 and c_1 were known since long ago [8]. The third integral c_2 found by Lidov allows to reduce the problem to the investigation of the behavior of integral curves in the plane (ω, e) with fixed c_1 and different values c_2. The qualitative analysis of integral curves has resulted in interesting celestial mechanics conclusions as follows: 1) the satellite orbits with constant e and ω exist in the averaged problem. This special point with the libration change of ω in its neighborhood is called often "Kozai–resonance" by the name of Kozai who investigated a more general case of the problem one year later [1]. Since the qualitative peculiarities revealed by Lidov remain valid in this case the name "Lidov–Kozai–resonance" as proposed by A. I. Neishtadt would be justified; 2) in particular case $c_1 = 0$ when the satellite orbital plane is orthogonal to the orbital plane of the perturbing body the evolution leads to transforming the orbit into the straight line. For small values of c_1 the orbital evolution for any initial values of ω involves a strong increase in e. As the semi–major axis $a = c_0$ is constant the distance of the pericenter becomes equal to the planetary radius and satellite falls onto its surface. This peculiarity of evolving orbits might be called "Lidov–Kozai–mechanism".

It's interesting to note that the above peculiarity of almost orthogonal satellite orbits conflicted with the real existence of the Uranus' satellites. Near circular and equatorial orbits of these satellites are inclined to the ecliptic plane by 98°. The analysis of real physical model taking into account the Uranus' oblateness resolved this contradiction and admitted the existence of circular orthogonal orbits [6]. Moreover, it leaded to the formulation of a new celestial mechanics problem (double–averaged Hill's problem taking into account the oblateness of central planet). For analyzing this problem two parameters are essential: γ — which characterizes the ratio of perturbing accelerations due to the planet's oblateness and external body; ϵ — the inclination of equatorial planet's plane to the orbital plane of the perturbing body. In general case this problem doesn't allow any more three first integrals in involution. Nevertheless, this problem has a number of integrable cases and particular solutions revealed in [7,3]. Of the most interest is the coplanar case ($\epsilon = 0$) investigated earlier by numerical way [2].

2. In what follows the researches carried out in the recent years and representing the development of Lidov's works are briely described.

2.1. In our investigations of the double–averaged Hill's problem taking into account the oblateness of the central planet there was established the invariance of evolutionary system with respect to a certain transformation of the elements and time. The families of the periodic solutions have been revealed and actually constructed (mainly numerically). These solutions correspond to the so–called periodically evolving satellite orbits characterized by the change of all four elements i, e, ω, Ω with the same period. The analysis of the stability of the stationary solutions has been carried out. In their small vicinity the symmetric and asymmetric periodic solutions have been constructed. The numerical prolongation of periodic solutions to the regions far from the stationary points permit to reveal their closed families. The regions of the stability of discovered periodic solutions have been determined by numerical way (see [9] and references therein).

2.2. The development of Lidov's techniques is also of much importance for the theory of motion of natural satellites. It concerns primarily the orbits of new satellites of the giant planets discovered from the end of 1997 to the beginning of 2000. They move in the regions, where the influence of the solar perturbation considerably exceeds the influence of the central planet's oblateness ($\gamma \ll 1$). For the analysis of their evolution in the first approximation it is possible to use the double–averaged Hill's problem in taking $\gamma = 0$. Qualitative investigation of this problem initiated by Lidov has been completed by us in 1999 with construction of the general solution depending on four arbitrary constants, i.e. initial values of the elements. For calculation of Ω an analytical expression in the form of elliptic integrals of the first and third kinds was received. Our subsequent papers have dealt with the qualitative peculiarities of the evolution of new satellite orbits. The simplest classification of these orbits was proposed together with the determination of the approximate characteristics of the evolution including the extreme values of e and i as well as the periods of the circulation of ω and Ω. In the framework of the employed double–averaged Hill's problem for four new Saturn's satellites the libration of ω was detected. However, it should be mentioned that their integral curves are located near to the separatrix. Therefore some libration orbits can pass to the group of circulator if the elements of the satellite orbits or the model of the evolution are defined more precisely. The above peculiarity is extremely rare even among ensemble of thousand of asteroid orbits. Therefore, the fact that among twelve orbits of recently discovered Saturn's satellites four orbits turn out to be potential librators is strange enough.

Another phenomenon in satellite systems of Jupiter, Saturn and Uranus is the distribution of the semi–major axes of the satellite orbits. In Jupiter's system the external satellites are divided evidently into two groups, direct and indirect ones. In Saturn's system there is the range of the semi–major axes where there exist both direct and indirect orbits. In the Uranus' system all discovered by today external satellites have indirect motion. Moreover, in all three systems there are the regions of the semi–major axes free from satellite orbits. The revelation of possible mechanism of "avoiding" above regions by the satellites presents a very interesting problem. In the Uranus' system such "emptiness" which separates internal and external satellites stretches approximately from 0.6 million km (Oberon's orbit) to 7 million km (Caliban's orbit), where the perturbing influence of the Sun is comparable with that of the oblateness of the central planet. A qualitative analysis of one of integrable cases of the double–averaged Hill's problem taking into account planet's oblateness performed by Lidov [7] allows to presume (as a hypothesis) the celestial mechanics explanation of the absence of the equatorial Uranus' satellites in the region $a > 1.3$ million km. If there were equatorial satellites in the above region, then their orbits should have begun to intersect with internal satellite orbits. In this case the probability of close approaches and collisions with internal satellites increases, as they were to go to more distant orbits or fall down onto the internal satellites filling up their masses essentially. Indirect confirmation of this fact is remarkable massivity of the internal Uranus' satellites as compared with external ones. More details may be found in [10] and references therein.

This study is supported by the Council of Grants of the President of Russia and the State support of Leading Scientific Schools (grant No. 00–15–96036).

References

1. Kozai Y. Secular Perturbations of Asteroids with High Inclination and Eccentricity. Astron J., 1962, **67**, 591–598.

2. Kozai Y. Motion of a Lunar Orbiter. Publs. Astron. Soc. Japan, 1963, **15**, 301–312.

3. Kudielka V. Balance Earth Satellite Orbits. Cel. Mech. & Dyn. Astron., 1994, **60**, 455–470.

4. Lidov M. L. The Evolution of Orbits of Artificial Satellites of Planets Under the Action of Gravitational Perturbations of External Bodies. Iskusstvennye Sputniki Zemli, 1961, **8**, 5–45 (in Russian).

5. Lidov M. L. The Evolution of Orbits of Artificial Satellites of Planets Under the Action of Gravitational Perturbations of External Bodies. Planet. Space Sci., 1962, **9**, 719–759.

6. Lidov M. L. On the Approximated Analysis of the Orbit Evolution of Artificial Satellites. In: Dynamics of Satellites (ed. M. Roy), Berlin–Gottingen–Heidelberg: Springer–Verlag, 1963, 168–179.

7. Lidov M. L. Integrable Cases of Satellite Problem with the Third Body and Oblate Planet. In: The Stability of the Solar System and Small Stellar Systems (ed. Y. Kozai), IAU, 1974, 117–124.

8. Moisseev N. D. On Some Basic Approximated Schemes of Celestial Mechanics Obtained by Averaging the Restricted Circular Three-Point Problem. 2. On the Averaged Variants of Three-Dimensional Restricted Circular Three-Point Problem. Proceedings of the Sternberg Astronomical Institute, 1945, **15**, 100–117 (in Russian).

9. Vashkov'yak M. A., Teslenko N. M. Stability of Periodic Solutions of Hill's Averaged Problem with Allowance for Planetary Oblateness. Astronomy Letters, 2001, **27**, 198–205.

10. Vashkov'yak M. A. Orbital Evolution of Jupiter's Outer Satellites. Astronomy Letters, 2001, **27**, 671–677.