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On convergence domains of expansions of disturbing function of the planetary three–body problem in powers of eccentricities

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The problem of convergence of the expansion of the disturbing function R in power series has long history [1–4]. The most intriguing point in these works is the fact that their results are different even for the plane case. Meanwhile as noted by Ferraz-Mello [5] "the importance of this subject in current research is increasing because of the possibilities open for the analytical study of the motion of planets and asteroids by new averaging techniques and by computational facilities for algebraic manipulation". He has analysed the results of [2, 4] in [5] and has shown that the convergence conditions by Sundman [4] are indeed necessary and sufficient, but that ones of Silva [2] are only sufficient. Using the steepest descent methods Ferraz-Mello has extended the Sundman criterion to the case of mutually inclined planetary orbits. He has established that the expansion of R for the action of Jupiter on an asteroid diverges for considerable number of asteroids in the outer region of the main belt, and hence their motions cannot be studied using expansion of this kind.

The above–mentioned authors have investigated the expansion of the principal part of R, Δ^{-1}, in power series with coefficients in terms of the mean anomaly l as an independent variable of the equations of perturbed motion. However, as it is known [6], the use of the eccentric, u, or true, f, anomaly instead of l may give some advantages because, for instance, it leads to the more rapid convergence of series in the two–body problem and allows to represent the coordinates of the perturbed planet through the elementary functions. The problem of the convergence of the Δ^{-1} expansions in powers of the eccentricities with coefficients as functions of u and f has been considered in [7,8], but only the divergence conditions have been given in them.

In the present paper a general method for determining the convergence domains of the R expansions in powers of the eccentricities with coefficients in terms of any anomaly, l, u or f, is suggested. The method is based on the general properties of the analytical functions [9]. The planetary configurations corresponding

to the greatest lower bound of the convergence domains have been found. It has been shown that the use of l and f gives, respectively, the least and the largest convergence domains for Δ^{-1}, but at the same time it gives the largest and the least ones for the indirect part of R. Advantages of using each anomaly are discussed.

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