

Superosculating intermediate orbits and their applications to study the perturbed motion

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A method of construction of intermediate orbits for approximating the real motion of celestial bodies in the initial part of trajectory is described. The method is based on introducing a fictitious attracting centre and is used to generalize Batrakov's approach [1]. The main idea of the method, proposed and realized in earlier publications [2,3] for the first time, lies in the fact that the gravitational parameter of the fictitious centre for the initial part of motion is not constant but is determined as a function of time from the condition minimizing the perturbations. The motion along the intermediate trajectory about the fictitious centre is not Keplerian. In the present work it is described by the equations of the perturbed version of the Gylden–Meshchersky problem. This generalizes our approach as compared with the previous papers [2,3], in which we have used the equations of the Gylden–Meshchersky problem in their classical form [4]. Here, we consider a case where the gravitational parameter defining the intermediate motion varies in accordance with the Eddington–Jeans mass–variation law. New classes of orbits having second– and third–order tangency to the perturbed trajectory at the initial instant of time are constructed. For planar motion, the tangency increases by one order. The constructed intermediate orbits approximate the perturbed motion better than the osculating Keplerian orbit and analogous orbits of other authors.

The applications of the orbits constructed in Encke's method [5] for special perturbations and in the procedure for predicting the motion in which the perturbed trajectory is represented by a sequence of short arcs of the intermediate orbits are suggested. The first application leads to generalized Encke's algorithms. It results also in new methods for solving the equations of orbital motion whose accuracy order coincides with the tangency order of the used intermediate orbit. The application of the Runge rule and Richardson extrapolation [6] to the latter allow us to obtain methods of higher orders. The new methods have been compared with the classical Encke method and numerical integration of the equations

of motion by the well-known Runge–Kutta–Fehlberg methods of the 4th and 7th orders [6]. The comparisons have been performed in computing the perturbed orbits of some small bodies of the Solar system.

The constructed intermediate orbits can be effectively applied also in the problem of determining preliminary orbits of celestial bodies from observations and their subsequent improvement.

References

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