IAA Transactions, No. 8, "Celestial Mechanics", 2002

Comparison of conditionally periodic solutions with the results of numerical integration in the two rigid body problem

N. Yu. Saburova

Arkhangelsk State Technical University, Arkhangelsk, Russia

In the papers [1], [2] the general methods of the proof of existence and construction of conditionally periodic solutions of canonical systems of differential equations have been proposed.

These methods have been applied in celestial mechanics for constructing stationary and conditionally periodic solutions in such problems as the restricted three–body (material points) problem [4], the problem of motion of a material point in a gravitational field of a rigid body (triaxial ellipsoid, geoid) [4], the problem of translational–rotational motion of a satellite in a gravitational field of a triaxial planet [5]. The magnitude characterizing the difference of the studied bodies from spheres and the ratio of the linear sizes of the bodies to the distance between them have been taken therewith as small parameters.

In the present paper conditionally periodic solutions in the problem of translational–rotational motion of two rigid bodies are constructed. The equations of motion are written in Delaunay–Andoyer canonical variables. The expansion of the force function of the interaction of two rigid bodies into Fourier series in angular variables obtained by Sidlichovsky [3] is used. The coefficients of the expansion depend on the Stokes constants of the bodies.

As a small parameter μ we take the magnitude characterizing the difference of the studied bodies from axial symmetrical ones in the Duboshin case admitting the Keplerian motion. The sufficient conditions of existence of the Keplerian motion in the Duboshin case are found. For elimination of the short periodic perturbations the new canonical variables are introduced instead of Delaunay–Andoyer ones. It is shown that the averaged system (after eliminating of the short periodic terms) has stationary solutions in neglecting the terms of the second order and higher with respect to the small parameter μ in the averaged Hamiltonian. The angular variables of the stationary solutions of the problem satisfy certain conditions resulting from some relationships between the position variables. The

obtained results are represented as the diagram of the dependence between the semi–major axis of the orbit and the eccentricity. On the basis of the found stationary solutions the conditionally periodic ones in the two rigid body problem are constructed. For this purpose an explicit form of the generating function of canonical transformation is found.

Numerical integration of the equations of translational–rotational motion of two rigid bodies is performed for a particular case. The obtained results are compared with the conditionally periodic solutions found by analytical techniques.

The work is supported by RFBR(00-02-17677a).

References

1. Grebenikov E. A., Riabov Iu. A. New Qualitative Methods in Celestial Mechanics. M.: Nauka, 1971 (in Russian).

2. Grebenikov E. A., Zhuravlev S. G., Riabov Iu. A. Conditionally Periodic Solutions of Canonical Systems of Differential Equations at a Sharp Resonance. ITEP-23, Moscow, 1976 (in Russian).

3. Sidlichovsky M. The Elimination of the Short Periodic Perturbations in the Problem of Two Finite Bodies. Bull. Astron. Inst. Czechosl., 1981, **32**, 159–167.

4. Zhuravlev S. G. Method of Investigation of Sharp-Resonant Problems of Celestial Mechanics and Astrodynamics. Arkhangelsk: "SOLTI", 2000 (in Russian).

5. Zlenko A. A. Construction of Conditionally Periodic Solutions in the Problem of the Translational–Rotational Motion of an Axisymmetric Satellite of a Triaxial Planet. Kosmicheskie Issledovaniia, 1984, **22**, 531–536 (in Russian).