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High–accuracy modelling of the Earth's gravitational potential from GOCE satellite gradiometry mission

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The Earth's gravitational potential is presented in form of a spherical harmonic series with the coefficients $\overline{C}_{n,m}$ which are the fundamental dynamical constants for the Earth. The set of $\overline{C}_{n,m}$ for degree $0 \leq n \leq N$ and order $|m| \leq n$ constitutes a geopotential model. A global geopotential model is a valuable source of information about the nature and composition of our planet. It is strongly needed for solving a large area of problems in satellite dynamics, geodynamics, oceanography, navigation, for creating a universal height reference system, etc. The basic data for constructing the geopotential models are provided by the measurements of the terrestrial gravity anomalies. However these data cover only one third of the Earth's surface. The application of perturbations of satellite orbits reached its intrinsic limits: only the potential harmonics up to degree and order 70 can be recovered by this approach while the contemporary demands are at least for 360. The reason for it is the presence of damping factor $q_n = (\frac{R}{r})^n$ at the potential harmonic of degree n where R and r are the Earth's mean radius and the satellite geocentric distance, respectively.

A great progress in modelling the Earth's potential V is expected only from satellite gradiometry missions – measuring the gravity gradient components. They represent six second order potential derivatives in the local reference frame centered at the satellite. In the expansions of these derivatives the n-th harmonic has an additional factor n^2 which partly compensates the attenuation effect of the factor q_n in the series for V. The first dedicated satellite gradiometry mission GOCE (Gravity field and Ocean Circulation Explorer) will be in the context of the European Space Research Agency (ESA) program. It will allow to construct a geopotential model whose accuracy and resolution is one order higher than for the present models.

The problem of recovering the geopotential coefficients from satellite gradiometry data can be easily solved if there are linear relations between the spectral coefficients of the observables and the coefficients $\overline{C}_{n,m}$ of V. However such relations have not been derived yet because the geopotential derivatives with respect to the north–oriented coordinates centered at the satellite contain singularities at the poles. For this reason, instead of the simple spherical functions, the pure–spin tensor harmonics are proposed for application. Due to very complicated structure of these basic functions, such analytical approach is extremely difficult for practical constructing the geopotential models and only the numerical approach is developed by the international scientific consortium. However this approach will be also very problematic for the implementation since it assumes the simultaneous least squares processing of a huge amount of data (about 100 millions of observations) and a great number (90 000) of unknown parameters.

In the present paper the authors managed to solve the problem of the geopotential modelling from GOCE satellite mission by an analytical approach, on the basis of the conventional spherical functions. First a procedure is elaborated for removing the singularities in the expressions of the geopotential derivatives. Then simple basic analytical relations are derived for the first time between the spectral coefficients of six GOCE observables and the unknown geopotential coefficients $\overline{C}_{n,m}$. The derived relations can be used for solving two (inverse) problems. So long as GOCE mission is still in a preparation stage (until 2004), the spectra of the geopotential derivatives are generated at the height $h = 250\,km$ of GOCE satellite from the most advanced EGM96 geopotential model constructed by NASA on the basis of a great number of the satellite and terrestrial data. Then from the simulated spectra of GOCE observables the corresponding 'output' geopotential model is recovered from the same basic relations. The 'input' (EGM96) and 'output' (recovered) geopotential models fully coincide. Thus the derived basic relations allow to perform the whole simulation experiment on the geopotential modelling from GOCE mission with the estimation of different kinds of errors.

A number of numerical experiments are carried out on the basis of EGM96 geopotential model. At first the basic quantities, degree variances, are evaluated for GOCE observables. They represent the mean square of the n-th degree harmonic for each geopotential derivative. Very interesting specific regular behavior of the spectra of six second derivatives is revealed.

The elaborated procedure of modelling the gravitational potential for the Earth can be applied for constructing the expansions of the third and higher order geopotential derivatives referred to the reference frame centered at the satellite, which will allow to discover new properties of the Earth's gravitational field. The present analytical results can be readily utilized for modelling the gravitational fields of other planets and the Moon.