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Precision increase for the theory of relative motion of a satellite's pair

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Modern higher–precise theories of motion of special Artificial Earth Satellites (geodetical, navigational) consider all forces which one may describe with at least 30% precision, including so exotical ones as a reaction of photon flux due to radiowave–interchange with ground stations. Accuracy may be increased drastically when we confine ourselves by the description of *relative* motion of a pair of satellites close to one another. Here we regard similar satellites moving along the same track, i.e. the set of subsatellites points. The physical reason for accuracy increase is rather simple: both satellites undergo the same perturbations with a little shift in time only.

Let pass to more strict formulations. Denote $\mathbf{F}_1$ forces per unit mass depending on position and velocity of a satellite with respect to the solid Earth, but not on time explicitly. Such are attraction of the solid Earth in Newtonian and relativistic approach, Lorentz force, main part of the air drag. Let $\mathbf{F}_2$ are all other forces which we consider as disturbing ones. At the first step we may neglect them. Then in the frame related with the solid Earth we have differential equations of motion of the form

$$\ddot{\mathbf{r}} = \mathbf{F}_1 + \mathbf{J}\,, \tag{1}$$

$\mathbf{J}$ being the sum of centrifugal and Coriolis forces of inertia independent of t as well.

Denote $\mathbf{r}_1(t)$ a solution of (1). Due to the autonomy of (1) $\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$ is also a solution for any constant shift τ. Solutions $\mathbf{r}_k(t)$ correspond to a pair of satellites moving along the same track. The vector $\mathbf{r}(t) = \mathbf{r}_2(t) - \mathbf{r}_1(t)$ describes relative position of the second AES with respect to the first one.

Note that the centrifugal force and especially Coriolis one are not very small in comparison with the main part of the Earth's attraction. That is why one uses inertial frame as a rule. Denoting corresponding vectors by Greek letters we have

$$\boldsymbol{\rho}_k(t) = \mathcal{A}(t)\mathbf{r}_k(t)\,, \tag{2}$$

$\mathcal{A}$ being orthogonal matrix describing the Earth's rotation.

Finally,

$$\boldsymbol{\rho}(t) = \mathcal{A}^{-1}(\tau)\left[\boldsymbol{\rho}_1(t+\tau) - \boldsymbol{\rho}_1(t)\right] + \left(\mathcal{A}^{-1}(\tau) - \mathcal{E}\right)\boldsymbol{\rho}_1(t)\,, \tag{3}$$

$\mathcal{E}$ being a 3×3 unit matrix.

In case τ is small so are the differences $\boldsymbol{\rho}_1(t+\tau) - \boldsymbol{\rho}_1(t)$ and $\mathcal{A}^{-1}(\tau) - \mathcal{E}$. For testing the smallness it is better to use dimensionless quantities $n\tau$ and $n_0\tau$ equal to multiplied by τ satellite's mean motion and angular velocity of the Earth's rotation, respectively.

The expression in brackets in the formula (3) represents a small difference of close quantities. To avoid the loss of accuracy we may use Lie – series representation in powers of τ for the solutions of similar to (1) differential equations in the inertial space. In our case Lie – series in powers of τ begins with the first degree term. As to the second term of the sum (3), the difference of corresponding matrices has maximal modulus of eigenvalues equal to $2\sin(n_0\tau/2)$ and is also small.

We see that *errors in* $\boldsymbol{\rho}(t)$ *diminish by a factor of* $n\tau$ *or* $n_0\tau$ in comparison with the errors in $\boldsymbol{\rho}_1(t)$.

As to numerical estimations of corresponding errors they depend on the value of τ and, what is more important, on the satellite's orbit we choose. For close to the Earth's surface AES $n\tau$ plays the main role whereas for higher ones $n_0\tau$ does. As an example, for an orbit with the semi–major axis $a = 7400$ km and eccentricity $e = 0.01$ we have $n = 1.0 \cdot 10^{-3}\ \mathrm{s}^{-1}$ and $n_0 = 0.73 \cdot 10^{-4}\ \mathrm{s}^{-1}$. Choosing $\tau = 25$ s we have $n\tau = 0.025$ and simultaneously we guarantee that the distance between satellites lies in the interval from 160 to 200 km.

Note that for satellites higher than stationary ones the approach we deal with looses its advantages since the luni–solar disturbing forces become considerable. Meanwhile they depend explicitly on time.

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