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Regular and chaotic relative motion of a dumb–bell–shaped satellite

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In this paper an orbital two-body system consisting of two point masses m_1,m_2, connected with an ideally flexible massless inextensible thread is considered. Equations are obtained for the plane relative motion of the system on the taut thread (on the assumption that the centre of mass of the system moves along the Keplerian elliptic orbit). The condition of existence of such motion is derived. If this condition is not met the system moves freely without being constrained. The torques of gravitational forces, aerodynamic pressure, aerodynamic friction and the aerogradient effect are taken into account. Particular attention is focused on possible ways of motion chaotization.

First, the problem is investigated in case of taut thread and Keplerian elliptic motion of the centre of mass. Parameters of aerodynamic pressure a, aerodynamic friction b and the aerogradient effect k in these formulas are expressed in terms of aerodynamic coefficients C_1, C_2, masses m_1, m_2, the focal parameter of the orbit P, the density of the atmosphere at the perigee ρ_π and the atmosphere scale height H.

In case of circular orbit ($e = 0$) and in neglecting the aerodynamic dissipation ($b = 0$) the equation of motion has been solved and typical phase portraits have been considered and classified. It is characteristic of this case that aerogradient effect and dissipation together may cause the system to spin up with rather high (but finite) angular velocity ($\sim 2deg/sec$) corresponding to the so–called limit cycle of the second type, where the angular velocity is $\omega = \omega_0 R_0/H$, ω_0 and R_0 being the orbital angular velocity and radius of the orbit, respectively. When a phase trajectory meets zones of leaving the constraints the taut thread becomes slack and the motion is free which is described by different equations.

In a general case the free motion is bound to become connected. Further evolution of the motion depends on the character of the impact when the thread gets taut. For an absolutely elastic impact the energy of the motion is the same for all points of the trajectory.

For description of investigation of such kind of motion it is convenient to use the method of point mapping considering characteristics of the trajectory only at the time of impact. Examples of phase portraits obtained by this method are given. Corresponding to chaotic motion are areas filled with points. The areas are called a chaotic sea in which there are islands of regular (periodic and conditionally periodic) motion. Fixed points of the mapping correspond to the periodic motion.

Another way of chaotization (without impacts) is due to the nonautonomous equations of motion. This is the case when the orbit of the centre of mass is elliptic including the case of connected motion with a taut thread according to the previous equation of motion. Chaotization is due to the nonzero value of the eccentricity ($e = 0$). The effect of chaotization may be significant even for quite small values of eccentricity, since the density of the atmosphere has an exponential growth when the height increases, and the exponent is $\kappa = eR_\pi/H$, the ratio R_π/H is ~ 100. That's why even for $e \sim 0.01$ we have $\kappa \sim 1$.

Besides known general effects (chaotization appears near separatrices, islands of regularity) there are ones peculiar to the problem in question: there appear atolls of regularity (not only 'islands'). There is also a tendency of regularization for large values of aerodynamic parameter.

Chaotization of the connected motion and the limit mode are considered in the general case. All aerodynamic effects are taken into account including pressure, friction, aerogradient (and eccentricity of the orbit). Phase portrait was obtained as above by using the method of point mapping. It was found that the modes of motion tend to quasi-periodic rotation (in the vicinity of the limit cycle of the second type, which exists in case of circular orbit).

References

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