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High precision parallel algorithms of numerical integration of celestial mechanics problems

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We investigate different scenarios of dynamical chaos origin in Hamiltonian systems generated by some celestial mechanics problems. Numerical investigations were carried out of phenomena as follows:

- period doubling cascades [1];
- transversal intersections of hyperbolic point invariant manifolds [2].

Both above–mentioned phenomena require a big amount of processor time using high precision numerical methods of integration of systems of ordinary differential equations. There are some reasons for necessity of using such numerical methods. First of all, we use Poincaré mapping for searching, continuation and bifurcation analysis of periodic solutions of Hamiltonian systems. This method require a big amount of calculation at the first stage of its using. The second reason is that both phenomena require computation of orbits for a lot of revolutions.

Traditional PC-based computer complex has essential restriction on hardware bit grid (no more than 18 decimal digits) and hence we can define the coordinates of points on Poincaré section for long period orbits with accuracy no more than 10^{-15} and therefore we can obtain the other features of motion (such as multiplicators, Feigenbaum and scaling constants, homoclinic invariants, etc.) with essentially less accuracy.

We took as a base for constructing a high precision numerical method two very popular algorithms:

- variable–order extrapolation method of Gragg–Burlirsch–Stöer with variable–step described in [3];
- fixed–order variable–step implicit Runge–Kutta method by Everhart (up to 27 order) [4].

We compiled these algorithms to C programming language with free multiple precision library GMP (GNU MP library) [5]. This library supports high effective floating point arithmetic operations with arbitrary precision. Testing of high precision integrator for the planar Hill problem [1] showed that we could calculate periodic orbits with number of revolutions 1024 and more with accuracy no less than 10^{-25} and obtain the Feigenbaum and scaling constants with accuracy 10^{-10}.

For the reason of intensification of the computation process the programs were carried on a parallel cluster which was built with the set of PC computers under LINUX operating system with LAM/MPI software [6]. We created the parallel version of numerical algorithm for calculation of Poincaré map in MPI environment and obtained the reduction of computing time at the small cluster with 4 processors.

References

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