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On the orbits of extrasolar planets

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The discovery of planetary systems around alien stars is an outstanding achievement of science. Many new problems, connected with extrasolar planets, arise in astronomy. Main problems concern their formation. Discovered planetary systems differ sufficiently from the Solar system, and known theories of planet formation fail. Here we restricted ourselves to some dynamical problems. Many massive planets contrary to Jupiter have large eccentricities of orbits. We cannot observe small planets like the Earth. It is interesting to investigate possible orbits of such small planets. How ‘Jupiter’ with large orbital eccentricity affects the orbital evolution of other possible planets and stability of the system? The interesting problem is possible existence in this case of stable quasi–circular orbits similar to the Solar planet orbits. Another problem is to study the conditions and boundaries of stable, regular motions of small mass planet in systems having ‘Jupiter’ with large orbital eccentricity as well as properties of regular, stable, and irregular, chaotic motions. It may be that extrasolar planetary systems are less stable than the Solar system. For example, in [1] chaotic properties of planets motions in the system υ *And* are discussed.

We have investigated possible motion of small planet within the limits of restricted planar elliptical three–body problem by using numerical integration and analytical methods. Numerical integration is performed using codes developed by Krogh [2]. Main usual properties of regular motions of small mass planet obtained numerically are as follows: the semi–maior axis a is constant; in Lagrange variables $e\cos g, e\sin g$ the trajectories are approximately circular (e is the eccentricity, g is the pericenter argument); center of circles is located on abscissa axis.

This picture can be described analytically using classical Laplace–Lagrange theory of secular perturbations. After averaging over two fast variables we have $a = const$ and $\dot{x} = -K_1 y$, $\dot{y} = K_1 x - K_2 e_p$. Here $x = e\cos g, y = e\sin g$ are Lagrange variables,

$$K_1 = \frac{mn}{\pi}\left[\frac{1+\alpha^2}{(1-\alpha^2)^2}\mathbf{E}(\alpha) - \frac{1}{1-\alpha^2}\mathbf{F}(\alpha)\right],$$

$$K_2 = \frac{mn}{\pi\alpha}\left[\frac{2(1-\alpha^2+\alpha^4)}{(1-\alpha^2)^2}\mathbf{E}(\alpha) - \frac{2-\alpha^2}{1-\alpha^2}\mathbf{F}(\alpha)\right],$$

m is mass of 'Jupiter', its parameters denoted by index p, $n = a^{-3/2}$ is mean motion of small planet, $\alpha = a_p/a < 1$, the mass of the star and gravitational constant are equal to unity, **E** and **F** are elliptic integrals. We have the integral of motion

$$y^2 + (x - e_p K_2/K_1)^2 = const.$$

After simplification we derive $K_2/K_1 \approx 9\alpha/8$. We have circles with center on abscissa axis $x = 9\alpha e_p/8$. This result agrees with numerical data, accuracy is about 10%. For example, initially circular orbit reaches eccentricity $e_{max} = 9\alpha e_p/4$ at most. The orbit with initial values $e = 9\alpha e_p/8, g = 0$ has no evolution. The same picture corresponds to the case $\alpha > 1$. The evolution of distance between orbits is investigated.

If the motion is irregular (chaotic) in the case $\alpha < 1$, than the values a and e usually increase consistently under condition $a(1-e) \approx const$. Initially circular orbit transforms to that similar to the case of long–periodic comets.

The regular eccentricity evolution described above demonstrates that stable quasi–circular orbits of small planet are unusual. However, such rare orbits are found and investigated.

Regions of regular and chaotic motions are separated and described for different values of mass and orbital eccentricity of large planet. The instability of chaotic trajectories is investigated.

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References

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