IAA Transactions, No. 8, "Celestial Mechanics", 2002

Geometrical analysis of solutions of restricted three–body problem

V. I. Prokhorenko

Space Research Institute, Moscow, Russia

This investigation concerns the problem of the choice of the long living elliptical orbits with high eccentricity. The aim is to construct a suitable approach for this choice taking into consideration the evolution of the orbital parameters under the perturbing influence of the external bodies. For the qualitative analysis of the orbital evolution we use the solution of the restricted circular double–averaged three–body problem for $\alpha = a/a_k << 1$ obtained by M.L. Lidov [1] in 1961. This solution includes three first integrals:

$$c_0 = a, \qquad c_1 = \varepsilon \cos^2 i, \qquad c_2 = (1-\varepsilon)(2/5 - \sin^2 i \sin^2 \omega)$$

and the quadrature $t - t_0 = -2\pi a^{3/2}(\mu A)^{-1} L(c_1, c_2, \varepsilon_0, \varepsilon, \omega_0)$, where

$$A = (15/2)\pi (M_k/M)(a/a_k)^3 {\varepsilon_k}^{-3/2}\,;$$

$$L(c_1, c_2, \varepsilon_0, \varepsilon, \omega_0) = \int_{\varepsilon_0}^{\varepsilon} [(1-\varepsilon)\varepsilon^{1/2} \sin^2 i \sin 2\omega]^{-1} d\varepsilon\,;$$

a is the semi major axis; $\varepsilon = 1 - e^2$, e is the eccentricity; i, ω are the inclination and the argument of pericenter measured relative to the disturbing body orbit; M, M_k are the masses of the central and disturbing bodies; μ is the product of the M and the gravity constant f; a_k, ε_k are the parameters of the orbit of the disturbing body. The constants c_0, c_1, c_2 are determined as functions of the initial values of the orbital elements: $c_0 = a_0$, $c_1 = \varepsilon_0 \cos^2 i_0$, $c_2 = (1-\varepsilon_0)(2/5 - \sin^2 i_0 \sin^2 \omega_0)$.

The spherical coordinate system $Or\theta\lambda$, where $r = \varepsilon$, co–latitude $\theta = i$, and $\lambda = \omega$ has been proposed [2] for the geometrical analysis. The origin of the frame $Or\theta\lambda$ coincides with the central body S. The integral $c_1 = const$ corresponds in this frame to the revolution surface, while $c_2 = const$ corresponds to a closed line in this surface. The integral curves go around the separate point of the center type corresponding to $c_2 = c_{2\,\max}(c_1)$ in the case $c_2 > 0$. The parameter ω varies monotonously anti–clockwise with the period equal to 2π. Such kind of evolution is marked as a rotation. The changing of parameters ε and i is vice versa of oscillation character. In the case $c_2 < 0$ the curves go around one of the separate points corresponding to $c_2 = c_{2\min}(c_1)$, and $\omega = 90°$ or $\omega = 270°$. The evolution of the parameter ω is in this case of the libration type. The domains of the different types of the ω evolution are separated by the separatrices $c_2 = 0$. We use the similarity theory to present the non–dimensional period T_* of the evolution of the parameters ε and i as a product of the independent factors: $T_* = -(4/15)(a_*^{-3/2}/L_D)|L_c(c_1, c_2)|$. Here $L_D = \mu_{k*}a_{k*}^{-3}\varepsilon_k^{-3/2}\mu_*^{-1/2}$ is the non–dimensional parameter of disturbances similarity dependent on the dynamical characteristics of the central and disturbing bodies only; $L_c(c_1, c_2)$ is the double quadrature L calculated for the limits $(\varepsilon_{max}, \varepsilon_{min})$. The non-dimensional parameters, marked by the dot *, are expressed in terms of some character parameters: dimension l, time τ, and mass m. Let us use for $L_c(c_1, c_2)$ the name "configuration parameter of the orbit". The investigation of the behavior of this function of two variables helps to find some of latent regularities. Of most interest is the existence of the quasi–symmetric solutions in the regions of the different types of the ω evolution (libration or rotation). We mean the solutions with the equal absolute values of the configuration parameters, corresponding to c_2 of the same absolute values but of the opposite signs. We shall take into consideration the finite dimension of the central body to estimate the ballistic lifetime. The region of the satellite existence corresponds to the parts of the integral curves located outside the spherical surface $\varepsilon = \varepsilon*$, corresponding to the impact of the satellite with the central body. In accordance to [1] $\varepsilon* = (2a_* - 1)/a_*^2$, where $a_* = a/R$, and R is the radius of the central body. The non–dimensional majorizing function of the ballistic lifetime is $T_{B*} = -(4/15)(a_*^{-3/2}/L_D)|L_b(c_1, c_2, a_*)|$, where $L_b(c_1, c_2, a_*)$ is the double quadrature L calculated for the limits $(\varepsilon_{\max}, \varepsilon*)$ in case $\varepsilon_{max} > \varepsilon* \geq \varepsilon_{min}$. It should be mentioned that $|L_b(c_1, c_2, a_*)| < |L_c(c_1, c_2)|$ and $|L_b|$ has just the same mirror quasisymmetry relative to $c_2 = 0$ as the function $|L_c|$ has. The main result of this investigation is the creation of the geometrical tool for the suitable choice of the orbits taking into account the ballistic lifetime and the character of the orbital parameters evolution. The efficiency of this tool was checked by the retrospective analysis of the orbits of the AES PROGNOZ series, $(a = 16.7R_E)$ launched in 1972–1995.

References

1. Lidov M. L. Orbit Evolution of Artificial Satellites of Planets under the Action of Gravitational Perturbations of External Bodies. Iskusstv. Sputn. Zemli, 1961, No. 8, 5–45 (in Russian).

2. Prokhorenko V. I. A Geometric Study of Solutions to Restricted Circular Double–Averaged Three–Body Problem. Kosm. Issled., 2001, **39**, 583–593 (in Russian).