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Celestial Mechanics and the real Solar System: measurements, models and tests

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The equations of motion of the gravitational N-body problem are not just a model, they are a *perfect* model, *the* most succesful mathematical model of physical reality ever conceived by man. This has been true for more than 300 years, but we need to test if this is still true to the level of accuracy required today. That is, the model accuracy needs to match the accuracy possible today in the measurement of the positions of celestial bodies, both natural and artificial.

1. Progress in measurement technologies

Astrometry, the traditional method to measure the position of a celestial body, has progressed in accuracy by about an order of magnitude in the last decades, as a result of the use of CCD and other electronic technology, and as an effect of the improvement of star catalogues. Still the accuracy is typically between 0.5 and 0.2 arcsec for most observations, taken from the ground. The problem is reliability, weighing of the observations and accounting for bias and correlation [1].

Planetary radar has (since the 60s) allowed to measure the distance to natural bodies with accuracies of the order of 1 km, the main limitation still being the unknown topography [2]. Asteroid radar has achieved accuracies of a few tens of meters in range, by modeling the shape and rotation [3].

Tracking of interplanetary spacecraft currently achieves accuracies in range of tens of meters. The new multi-frequency technologies allow to measure range-rate to a few micron/s, range to tens of cm [4]. Thus an orbiter around a natural body will allow to improve the orbit to a level several orders of magnitude better than the measurements of the body itself [5]. The examples of the BepiColombo mission to Mercury [6] and of the Don Quijote mission to an asteroid [7] indicate a new class of achievable scientific goals.

2. Newtonian models

In a purely newtonian model, a small body (spacecraft, asteroid) orbits around the Sun under the attraction of a finite number of gravitational point sources (or spherically symmetric bodies). Correction for shape (e.g., J_2 of the Sun, of Jupiter) can be easily added when required, but for most attracting bodies this is not necessary.

The question for an N-body model is: how much is N? The Roy-Walker formalism [8] allows to easily compute the size of the (short and long term) perturbations introduced by adding one body to the model. If the perturbation is negligibly small, for numerical stability reasons it is necessary to remove the body from the model. The examples of the perturbations from the Galilean satellites of Jupiter, from the Moon, from Mercury are discussed.

For orbit determinations, based on the top accuracy tracking now possible, the critical issue is the perturbations by a large number of asteroids. This already sets the limit to the accuracy in the determination of the orbit of Mars. In future missions, perturbations to the Earth by asteroids could be a significant contribution to the error budget. Thus to determine the mass of the asteroids has become a scientific goal relevant outside the field of asteroid studies.

3. Non gravitational perturbations

Non gravitational perturbations are small because they are anyway surface forces, thus contain the area to mass ratio as small parameter. E.g., proposals of electromagnetic perturbations for large bodies are ridicolous. However, when the accuracy of orbit determination becomes extreme, the direct and indirect efects of radiation pressure could be significant at a measurable level.

The Yarkovsky effect is a form of thermal thrust, with the possibility of acting in a secular way on the semimajor axis of an asteroid, thus some of its effects accumulate quadratically with time. For small asteroids (in the km size range) well observed with radar, this effect can be measured; the 2003 observation campaign of (6489) Golevka has the possibility of achieving the first experimental confirmation of Yarkovsky effect on a natural body [9]. Direct radiation pressure could also be relevant for bodies with non-uniform albedo [10].

On longer time scales, the non gravitational perturbations can be enhanced by chaotic effects. The case of the asteroid (29075) 1950DA shows that our capability to predict an asteroid impact, over a time scale of centuries, does depend upon an accurate thermal model of a natural body [11]. This is not the case over the time span (a few decades) currently considered in impact monitoring systems.

4. Relativistic models of the Solar System

Relativistic celestial mechanics has been available for long time, and it is well known how to correct the equations of motion of the N-body problem at least to the Post Newtonian (PN) order v^2/c^2. Already planetary radar has allowed to constrain the PN parameters, that is to perform experiments in General Relativity (GR) by using observations of the orbits of the planets [2]. The new tracking technologies should allow to improve these results by two orders of magnitude, but this is far from trivial. Long term landers are too challenging technologically (e.g., on Venus and Mercury; Mars is too much perturbed by asteroids). An orbiting spacecraft hosting the transponder is affected by non gravitational perturbations. If these are measured by an accurate on-board accelerometer, then the orbit around a planet can be determined well enough to constrain the motion of the planetary center of mass, roughly with the same accuracy of the spacecraft tracking.

The BepiColombo mission around Mercury in the years 2010s will carry an ultra accurate Radio Science Experiment, with GR tests as one of the scientific goals [5]. The orbit of Mercury will thus be used to test the consistency of the β, γ PN parameters with GR, and also to test the Strong Equivalence Principle, to measure the J_2 of the Sun and some preferred frame parameters. This experiment has been fully simulated, but of course to handle the real data at these levels of accuracy will be a challenge for the next generation of celestial mechanicians.

5. Deflecting an asteroid

The most spectacular example of ultra accurate Celestial Mecahnics is the experiment, now under study by the European Space Agency, on how to deflect an asteroid, in case it was found on a collision course with our planet. A spacecraft impactor can deflect a sizeable asteroid (500 m diemeter) by a few tens of micron/s, and this is well measurable, provided an asteroid orbiter equipped with up to date tracking instruments and accurate accelerometers is available [7]. The technique to derive the asteroid orbit is reminiscent of the one proposed for the relativistic experiment based on Mercury.

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