Expansion of the Hamiltonian of the planetary three–body problem into Poisson series in all elements using Poisson series processor PSP

E. D. Kuznetsov[1], K. V. Kholshevnikov[2], A. V. Greb[3]

[1]Astronomical Observatory of Urals State University, Ekaterinburg, Russia
[2]Astronomical Institute of St. Petersburg State University, St. Petersburg, Russia
[3]Institute of Applied Astronomy, St. Petersburg, Russia

Investigation of evolution of a planetary system similar to our Solar one is one of the most important problems of Celestial Mechanics. The first step to solve it involves expansion of the corresponding Hamiltonian into Poisson series in all elements. In this work we consider the case of two planets (the Sun – Jupiter – Saturn). We plan to pass to the general case in the nearest future.

Let m_0, $\mu m_0 m_1$, $\mu m_0 m_2$ be the masses of the Sun, Jupiter and Saturn, respectively. Small parameter μ is set equal to 10^{-3}. Indices 1 and 2 for radius vectors, coordinates and orbital elements correspond to Jupiter and Saturn. We use Jacobian coordinates as best–fitting for our problem. As to osculating elements we deal with two systems of them.

The first system. Positional elements $(a - a^0)/a^0$, e, $\sin(I/2)$ are small and dimensionless. Angular elements $\alpha = l + g + \Omega$, $\beta = g + \Omega$, $\gamma = \Omega$ are expressed in terms of 'broken' angles. Here a and a_0 are the semi–major axis and its average values, e, I, l, g, Ω are eccentricity, inclination, mean anomaly, argument of pericenter, and longitude of ascending node, respectively.

The second system realizes simplifications due to the homogeneity of the perturbation function with respect to the semi–major axes. On the other hand, it has a deficiency, mixing a part of elements of all planets. Namely, we use z_s, e_s, $\sin(I_s/2)$, α_s, β_s, γ_s. Here for the first planet $z_1 = \omega_1^0/\omega_1 - 1$, for the s^{th} $(s \geq 2)$ planet $z_s = \omega_1^0\omega_s/(\omega_s^0\omega_1) - 1$, $\omega_s = \kappa_s a_s^{-3/2}$ being the mean motions, ω_s^0 being constants close to the mean values of ω_s, κ_s being gravitational parameters. In this system denominators arising in the process of analytical integration of equations of motion are extremely simple.

Let represent Hamiltonian h as a sum of the unperturbed part h_0 and the perturbed one μh_1: $h = h_0 + \mu h_1$. The first term depends on semi–major axes only $h_0 = -Gm_0m_1/(2a_1) - Gm_0m_2/(2a_2)$. The second term may be thought of as a constant factor having dimension of velocity squared and a dimensionless part h_2: $h_1 = (Gm_0/a_0)h_2$ [1].

The disturbing function h_2 may be developed into Poisson series $h_2 = \sum A_{kn}x^k \cos ny$ in positional x and angular y elements. Coefficients A_{kn} are found using the Poisson series processor PSP [2]. To decrease round–off errors the rational version of PSP is used.

To obtain the averaged Hamiltonian h up to the second order with respect to the small parameter it is necessary to take into account terms up to the order $k = 6$ for the positional elements and up to the multiplicity $n = 13$ for the angular elements [3].

Four variants of the expansion are constructed, two for each of the osculating elements system. One of the variants furnishes numerical values of A_{kn} corresponding to the system the Sun – Jupiter – Saturn. Other one furnishes their literal expressions depending on parameters of the system. The expansions with numerical data contain 55228 terms. The expansions with literal parameters contain 182744 terms for the first elements system and 183227 terms for the second one.

This work was partly supported by the RFBR grant No. 02-02-17516 and the Leading Scientific School grant No. 00-15-96775.

References

1. Kholshevnikov K. V., Greb A. V., Kuznetsov E. D. Development of Hamiltonian of Planetary Problem in Poisson Series in All Elements (theory). Astronomicheskij Vestnik, 2001, **35**, 267–272 (Solar System Research, 2001, **35**, 243–248).

2. Ivanova T. V. Poisson Series Processor PSP. Preprint No. 64 ITA RAS, St. Petersburg, 1997 (in Russian).

3. Kholshevnikov K. V., Greb A. V., Kuznetsov E. D. The Expansion of the Hamiltonian of the Two–Planetary Problem into a Poisson Series in All Elements: Estimation and Direct Calculation of Coefficients. Astronomicheskij Vestnik, 2002, **36**, 75–87 (Solar System Research, 2002, **36**, 68–79).