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Determination of rectilinear orbits

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The rectilinear unperturbed motion is limiting case of elliptic, parabolic and hyperbolic motions. It is known for all types of rectilinear motion the orbit is part of the line passing through the center of attraction. This fact allows to develop the common approach for determination of preliminary rectilinear orbits.

The methods of determination of preliminary rectilinear orbit by two observations without any limitations on intervals of time between them are presented in this paper.

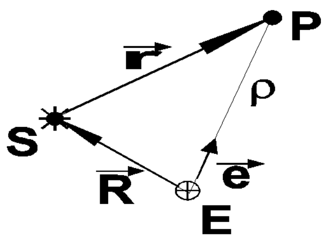
The orbit are determined by the method of Laplace, if the interval of time is small. This method is based on the use of the vectors of position and their derivatives. The condition of rectilinearity of orbit is:

$$\mathbf{r}\times\dot{\mathbf{r}} = 0, \tag{1}$$

where

$$\mathbf{r} = \mathbf{e}\rho - \mathbf{R}, \tag{2}$$

$$\dot{\mathbf{r}} = \dot{\mathbf{e}}\rho + \mathbf{e}\dot{\rho} - \dot{\mathbf{R}}, \tag{3}$$



S — the center of attraction,
E — location of observer,
P — the observated object,

e — unit vector of direction to object,
R — vector of center of attraction location with respect to observer,
r — vector of object location with respect to the center of attraction,

ρ — distance between observer and object.

The vectorial equation (1) is similar to the system of three scalar equations. After the substitution of (2) and (3) it is solved with respect of ρ and $\dot{\rho}$. Afterwards, we have found $\mathbf{r}$ and $\dot{\mathbf{r}}$ to determine the elements of object orbit [1].

The process of determination of orbit is separated into two stages. If the interval of time between the observations is large:
1) the determination of location of orbit in space by the geometrical method,
2) the determination of type of orbit and solving the equation of Euler–Lambert for finding the semi–major axis.

At the first stage, the line containing the required orbit is determined as intersection of two planes. It passes through the points S, P and E at moments of time t_1 and t_2, respectively. It is necessary that this line were not in the plane of motion of the observer with respect of the center of attraction. After the determination of line location, we determine the distances r_1 and r_2 at moments t_1 and t_2, respectively.

At the second stage, with the known time interval between the observations Δt and distances r_1 and r_2, we determine the type of orbit by comparison with the solutions for parabolic and border elliptic orbits [2] and solve the equation of Euler–Lambert for finding the semi–major axis.

References

1. Subbotin M. F. Introduction into Theoretical Astronomy. M.: Nauka, 1968, (in Russian).

2. Elyasberg P. E. Introduction into theory of flight of Earth's artificial satellites. M.: Nauka, 1965, (in Russian).