An improved analytical technique for accurate calculation of satellite motion perturbations due to the Moon/Sun/planets

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An improved analytical technique for accurate calculation of satellite orbit perturbations caused by attraction of the third body (the Moon, the Sun, planets) is presented. The work expands previous studies [1–3] of the author on development of accurate analytical theory of satellite motion. The theory is based on Poincare method of small parameter and enables one to calculate all perturbations proportional up to and inclusive the 5th-order of the parameter.

One of the major tasks arising in analytical methods of calculating effects of the Sun, the Moon, and planets attraction in satellite motion is an accurate representation of the third body orbits. In the simplest case the latter are assumed to be Keplerian ellipses with constant (or some secularly precessing) elements. More advanced studies employ some known well–developed analytical expansions of spherical coordinates of the Moon, the Sun, and planets and transform them to relevant series representing the perturbing functions of the bodies. However, the accuracy of this approach is obviously limited by accuracy of the original analytical expressions for the third body coordinates. So, some modern analytical theories use precise numerical planetary/lunar ephemerides of DE–series [4] as either an additional (see, e.g. [5]) or the main (in work [6]) source of the third body coordinates, but do that over a relatively short time interval (a few days only). In the present study we use DE–ephemerides for expanding the perturbing function due to the Moon, the Sun, and planets over long–term intervals (up to two thousand years).

After some simple modification of results obtained in [7, 8], the perturbing function due to the third body attraction can be presented as follows:

$$R = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} \left(\frac{a}{R_s}\right)^l \bar{F}_{lmp}(i) X_{l-2p+q}^{l,l-2p}(e) \times$$
$$\times \left(\bar{A}_{lm} \cos \psi_{lmpq} + \bar{B}_{lm} \sin \psi_{lmpq}\right)$$

where $a, e, i, \Omega, \pi, \lambda$ are osculating Keplerian elements of a satellite orbit, $\bar{F}_{lmp}$ is the normalized inclination function, $X_{l-2p+q}^{l,l-2p}$ is the Hansen coefficient,

$$\psi_{lmpq} = (l-2p+q)\lambda - q\pi + (m+2p-l)\Omega,$$

$$\bar{A}_{lm} = \begin{cases} \bar{C}_{lm} & \text{if} \quad l-m \quad \text{even} \\ -\bar{S}_{lm} & \text{if} \quad l-m \quad \text{odd,} \end{cases} \qquad \bar{B}_{lm} = \begin{cases} \bar{S}_{lm} & \text{if} \quad l-m \quad \text{even} \\ \bar{C}_{lm} & \text{if} \quad l-m \quad \text{odd,} \end{cases}$$

$$\bar{C}_{lm} = \frac{1}{2l+1} \sum_j \frac{\mu_j}{R_s} \left(\frac{R_s}{r_j} \right)^{l+1} \bar{P}_{lm}(\sin \delta_j) \cos m\alpha_j,$$

$$\bar{S}_{lm} = \frac{1}{2l+1} \sum_j \frac{\mu_j}{R_s} \left(\frac{R_s}{r_j} \right)^{l+1} \bar{P}_{lm}(\sin \delta_j) \sin m\alpha_j,$$

and $\mu_j, r_j, \alpha_j, \delta_j$ are, respectively, the gravitational parameter, geocentric distance, right ascension and declination of the j^{th} perturbing body (referred to the Earth mean equator and equinox of a fixed epoch, e.g. of J2000); $\bar{P}_{lm}$ is the normalized associated Legendre function; R_s is an arbitrary scaling parameter (in our study we chose the latter equal to 43000 km to have the ratio a/R_s being less than 1 for all Earth's artificial satellites at orbits extending up to the geostationary orbit, inclusively).

It is seen that the coefficients $\bar{C}_{lm}, \bar{S}_{lm}$ accumulate all information about instant positions of the perturbing bodies. By using the latest long–term ephemerides DE/LE–406 we calculated numerical values for those coefficients over the two thousand year interval [1000AD, 3000AD] with a sampling step 1 day. As the perturbing bodies we considered the Moon, the Sun, Venus, Jupiter, Mars, and Saturn. Then we made a spectral analysis of the calculated series by using the improved technique [9]. The feature of this technique is that the final expansions are Poisson series whith the arguments being high–degree polynomials of time as opposed to the classical Fourier analysis where arguments are always linear functions. The frequencies of the series are linear combinations of the fundamental arguments of motion of the Moon, the Sun, and planets. It results in the essential improvement in accuracy of the final series. In the spectrum of $\bar{C}_{lm}, \bar{S}_{lm}$ we took into account all "waves" of amplitude increasing the absolute level of $10^{-6}\ \mathrm{m}^2/\mathrm{sec}^2$ (the corresponding relative limit is about 10^{-8}). Expansions of the coefficients of up to degree $l = 8$ have been made. The total number of terms in the final expansion of the perturbing function over two thousand years is about 30000. The exact number of terms to be taken into account when calculating the third body perturbations depends on the satellite altitude and prediction time span.

The final series for $\bar{C}_{lm}, \bar{S}_{lm}$ are well adapted for the use in analytical calculation of the third body perturbations of satellite motion by means of computer.

We employ this technique in our analytical method for prediction of both high–altitude and low–altitude Earth satellites orbits, and the obtained results are presented as well.

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