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Families of periodic solutions of the planar restricted three–body problem and their application at designing the orbit for the space radio telescope

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1. The international project **"RadioAstron"** [1] provides to launch the space radio telescope with 10–meter antenna and to create together with the global ground VLBI network a unified system of the ground–space interferometer with capabilities, approximately in 10 times exceeding capabilities of ground interferometers. For implementation of this purpose it is required to design regularly evolved (under the action of the Moon) orbit with radius of apogee ~ 400 thousand km and with the greatest possible rotation of its plane and apsides line.

2. The periodic solutions of autonomous Hamiltonian systems [2, 3] belong to families of the periodic solutions with variable period and index of stability. For the stable solutions the transformation of the neighbourhood of the solution for period is reduced to turning through some angle φ. If φ rationally expresses in terms of 2π, $\varphi = 2\pi \cdot p/q$, p and q are integers, $q > 1$, the initial family with period T is intersected with the family of the periodic solutions with period qT. Poincare [2] has defined such solutions as **the periodic solutions of the second kind. The monodromy matrix allows to define the directions of prolongation both of initial family, and of a family, induced by it as the solutions of second kind**. If there is no second integral at $q > 4$ in the neighbourhood of this point of the family of the solutions of second kind there must exist [3] stable solutions **generating their own families of the periodic solutions of second kind and so ad infinitum**.

3. Practically all known periodic solutions of the restricted three–body problem are **symmetric with respect to axis** x_1. It involves that the equations of motion admit an invariant change of variables. Therefore, the monodromy matrix has a special form; it is possible to show, that **at the absence of the resonance 1:1 the prolongation of symmetric periodic solutions of this problem**

gives only symmetric periodic solutions. It is essential that **this statement is valid not only for prolongation of the initial family of periodic solutions, but for a family, generated by it as solutions of second kind**.

In case of a resonance 1:1 it is necessary to take into account the structure of elementary divisors of the monodromy matrix. In correspondence with the divisor $(\rho - 1)^2$ the monodromy matrix has two eigenvectors. One gives the direction of prolongation of the family of symmetric solutions with variable energy and duration of period. Second vector gives the direction of prolongation of new family of the solutions of second kind, for which energy and duration of period are constant (in the first approximation); **these solutions always are asymmetric**. At two other structures of elementary divisors the solutions remain symmetric.

4. In the restricted three–body problem there exist families of periodic (in rotating coordinate system) solutions around collinear equilibrium points. For motion in these orbits Keplerian elements change with a very high speed. If we select orbital parameters so that it would get in the neighbourhood (in four–dimensional space of coordinates and velocities) of orbits of one of these families, the motion will coincide with one around the equilibrium points for some time and Keplerian elements also will strongly change. The implementation of this idea has allowed to construct families containing periodic solutions with any rotational displacement (for one year) of apsides line.

5. The analysis of orbits obtained with the above described algorithms, has allowed to select a direction of searching real orbits which are met the requirements of the project **"RadioAstron"**. One of such orbits is described in detail in [1].

References

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