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Symplectic integrators for studying the long-term evolution of high-eccentricity orbits

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Discoveries of many new objects moving on high-eccentricity orbits in the Solar system and beyond have highlighted the importance of modeling the long-term evolution of such orbits. At present, symplectic integrators introduced by Wisdom and Holman [1] and Kinoshita et al. [2] are the most popular tools for studying dynamics of solar and extrasolar system objects. Mikkola and Tanikawa [3] and Preto and Tremaine [4] suggested a new integrator in which the time-step depends on the potential energy of a Hamiltonian system. In this paper we develop these methods to handle both high-eccentricity orbits and close encounters for the Hamiltonian of the form

$$H = H_0 - H_1,$$

where H_0 is the Keplerian part, H_1 is the perturbation part, and $H_0 \gg H_1$ in the absence of close encounters.

For the motion of a small body with infinitesimal mass in the gravitational field of the Sun and planets, $H_0 = v^2/2 - \mu/r$ is the Keplerian part, $H_1 = R(q_1, q_2, q_3, t)$ is the perturbing function, μ is a positive constant, $r = \sqrt{q_1^2 + q_2^2 + q_3^2}$, $v = \sqrt{p_1^2 + p_2^2 + p_3^2}$, and the conjugate canonical variables q_1, q_2, q_3 and p_1, p_2, p_3 are Cartesian coordinates and their time derivatives. The perturbing function R depends on t through planetary coordinates. We extend the phase space, adding the canonical variables $q_4 = t$ and $p_4 = -H$ [5]. Then there exists a transformation to the new independent variable s and the new Hamiltonian

$$\Gamma = \Gamma_0 + \Gamma_1,$$

where

$$\Gamma_0 = \log K_0, \quad \Gamma_1 = -\log K_1,$$

$$K_0 = r(H_0 + p_4 + B_0 + \frac{B_1}{r} + \frac{B_2}{r^2}), \quad K_1 = r(H_1 + B_0 + \frac{B_1}{r} + \frac{B_2}{r^2}),$$

$\Gamma = 0$ along the trajectory, B_0, B_1, B_2 are small constants, and

$$ds = \frac{K_0}{r}dt = \frac{K_1}{r}dt = (R + B_0 + \frac{B_1}{r} + \frac{B_2}{r^2})dt.$$

Both Γ_0 and Γ_1 are integrable. Therefore, the generalized leapfrog scheme[1] can be applied to the Hamiltonian Γ. The step over s is constant at the symplectic integration. Thus the time–step depends on the perturbing function R and the distance r. The practical choice of B_0, B_1, B_2 is considered for both barycentric and heliocentric coordinate systems in details. Although numerical tests have shown that the method is the most stable at $B_0 = 0$ for sufficiently small steps, the parameter $B_0 = 0$ can be used to keep the time–step within a small fraction of the shortest period in the dynamical system. In particular, the algorithm described above has been applied to integrations of trans-Neptunian objects for the age of the Solar system.

The same principles can be implemented for the general N-body problem. The extension of the algorithm to the Jacobi and mixed–centre coordinates [6] has been carried out. Numerical experiments demonstrate the efficiency of this symplectic technique for planetary system formation problems.

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