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Dynamics of resonant planets: The resonances 2:1, 3:2 and 5:2

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This paper considers the planar problem of two planets orbiting a star with periods close to a commensurability. The problem is stated in Hamiltonian form, in heliocentric canonical variables, and is reduced to two degrees of freedom through the elimination of all non–critical terms involving the mean longitudes. In the adopted approximation, the reduced Hamiltonian includes the main resonant and secular terms up to fourth order in the eccentricities. In the case of first–order resonances (Callegari Jr. et al, 2002) the second–order critical terms are also kept in the model. Numerical tests have shown good agreement between the solutions of the model and the solutions of the correspondent exact equations of a two–planet system, at least as far as the eccentricities are kept small. In the domains studied here, the eccentricity of the planets generally remains below 0.05 in the resonances of higher α, reaching 0.1 only in those of lower α (α is the ratio of the semimajor axes of the 2 planets.)

The dynamics of the resulting system can be studied through the construction of surfaces of section for a large set of initial conditions (eccentricities and critical angles). Besides, the FFT spectrum of all solutions is calculated and the number of peaks above a suitably chosen limit is used to determine the complexity of the solution (see Michtchenko and Ferraz-Mello 2001a). The results from the FFT spectrum are complementary to the surfaces of section in the study of the dynamics. As the Hamiltonian has two degree of freedom, two main independent frequencies of the system are expected to be found in the Fourier spectrum, as well as higher harmonics and beats of both frequencies. In this way, we may class the solutions as regular when the FFT spectrum shows a small number of harmonics (peaks) and chaotic when the structure of the FFT spectrum becomes complex and the number of peaks exceeds a given limit.

These two techniques were applied to three real 2-planet systems whose members have semimajor axes such that they are in resonance and are close to some important near resonant systems:

(a) the system Sun-Jupiter-Saturn, near the 5:2 resonance. ($\alpha \sim 0.54$);

(b) the system Sun-Uranus-Neptune, where both planets orbit the Sun near the 2:1 resonance ($\alpha \sim 0.63$);

(c) the system formed by the pulsar PSB B1257+12 and its planets B and C, near the 3:2 resonance ($\alpha \sim 0.763$).

In order to explain the results obtained on the dynamics of these systems let us define the critical angles. Let λ_i, ϖ_i, denote the mean longitudes and the perihelion longitudes of the inner (i=1) and outer (i=2) planets, respectively. The critical angles are $\sigma_i = (p+1)\lambda_2 - p\lambda_1 - \varpi_i$ $(i = 1, 2)$ ($p = 1$ for the 2:1 resonance, $p = 2$ for the 3:2 resonance and $p = 2/3$ for the 5:2 resonance. The long period (secular) variable is $\Delta\varpi = \sigma_1 - \sigma_2$.

The systems are studied following the technique proposed by Tittemore and Wisdom (1988) and uses two parameters: the energy and one parameter δ measuring of the distance of the system to the exact resonance (which is equal to zero at the exact resonance). This parameter is defined in terms of constants appearing in the expression of the Hamiltonian. It is roughly equal to $5n_2 - 2n_1 - \dot{\varpi}_1 - 2\dot{\varpi}_2$ in the study of the 5:2 resonance and $(p+1)n_2 - pn_1 - \dot{\varpi}_1$ in the case of the first-order resonances.

In the 5:2 resonance it was possible to identify several different regimes of motion. Outside the resonance, the solutions are the well-known secular ones. The surfaces of sections show 2 periodic orbits, named I and II by Michtchenko and Ferraz-Mello (2001b), where we have, respectively, $\delta\varpi = 0$ and $\delta\varpi = \pi$. These two periodic orbits correspond to four possible stable geometrical configurations of the planets:

- Mode I: the perihelia of the inner and outer planets are aligned in the same direction (parallel apsidal lines) ($\delta\varpi = \sigma_1 - \sigma_2 = 0$).

- Mode II: the perihelia of both planets are aligned in opposite directions (anti-parallel apsidal lines), ($\delta\varpi = \pi$).

In the lower energy surfaces of section, the solutions appear divided in two modes (mode I and mode II, respectively) formed by motions over two seemingly regular families of tori enclosing the 2 periodic orbits. The classical classification of the motions according with the behavior of the angle $\delta\varpi = \sigma_1 - \sigma_2$ leads to some difficulties well-known in the study of secular dynamics, because the periodic orbits are not at $e = 0$. Then, solutions on tori enclosing the origin and those on tori which do not enclose the origin are kinematically different, notwithstanding the fact that a bifurcation between them does not exist. The torus going across the origin, which separates the two kinematically different motions, is not a true topological separatrix. (Regular tori in both sides of this separatrix belong to a same regime of motion and form a continuous family.)

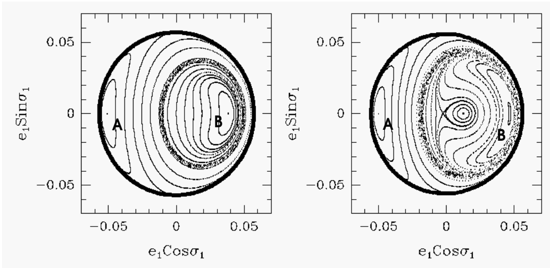

Fig. 1. Two surfaces of section of the 5:2 resonance showing the preserved secular mode I (A) and the main resonant modes R_{ex} and $R_{in}(B)$. Taken from Michtchenko and Ferraz-Mello, 2001b.

The solutions in modes I and II are such that the angle $\delta\varpi$ librate about 0 or π as far as these solutions are close to the periodic orbits. Solutions on wider tori may show this angle circulating, but this difference is topologically meaningless. Besides, for lower energies, we have to distinguish the solutions corresponding to systems in both sides of the resonance. When the ratio of the semi-major axes of Jupiter and Saturn is larger than the exact resonant value 0.54, the angles σ_i have retrograde motion (negative velocity) and when α is smaller than 0.54, the orbits of Jupiter and Saturn are closer one to another, the angles σ_i have direct circulation (positive velocity).

As the energy increases, the system reaches the resonance domain and the solutions of mode II located in the immediate neighborhood of the period orbit starts librating (both angles σ_i librate – a true bifurcation is reached). The new regime of motion is named R_{ex} (see the figure). The chaoticity of the motions separating librating and circulating orbits is clearly seen in the surfaces of section (the thick curves separating the domains A and B), and confirmed by the FFT of the solutions. For energies yet larger (right-hand side section in the figure), a secular resonance associated with the relative motion of the perihelia is reached creating a new bifurcation inside the libration region and the surface of section shows a new center and a new saddle point. The separatrix emanating from the saddle point encloses a new regime of motion (R_{in}), in the immediate neighborhood of a new stable periodic orbit. The sections were done for a given value of δ (0.02), but the regimes of motion reproduce themselves without modifications for other values of $\delta > 0$.

In the case of the Uranus-Neptune-like system the situation is more involving. Studies done with $\delta = 0.0012$ first show a behavior very similar to that of the resonance 5:2. Far of the resonance, the system is dominated by secular interactions showing the two regimes of motion (modes I an II) around stable periodic orbits; as the energy increases, the exact resonant separation of the two planetary orbits is reached and the solutions of mode II located in the immediate neighborhood of the period orbit starts librating (mode R_{ex}); for energies yet larger, a secular resonance associated with the relative motion of the perihelia is reached creating a new bifurcation inside the libration region and the surface of section shows a new center and a new saddle point. The separatrix emanating from the saddle point envelopes a new regime of motion in the immediate neighborhood of the new stable periodic orbit (mode R_{in}). However, from this point on, new regimes of motion, not seen in the 5:2 resonance, appear. While in the 5:2 resonance larger energies came to correspond to a deep resonance regime, very regular, in the 2:1 resonance new bifurcation appear in the surface of section. Another feature seen in the surfaces of section of the 2:1 resonance is that the chaotic zone separating the domains A and B in the figure grows and evolves in such a way to completely envelop the domain A. Periodic orbits at the center of the resonant modes may show either $\delta\varpi = 0$ or $\delta\varpi = \pi$. (The two angles σ_i may be librating on the same side or in opposite sides.) These centers migrate and in some cases they may cross the origin and change by 180° the libration center of one of the σ_i. The regimes of motion characterized by the coupled libration of σ_1 and σ_2 around 0, and libration of $\Delta\varpi = \pi$ have the same characteristics of the motions of the two planets orbiting around the star GJ 876 (Marcy *et al.* 2001).

The results for 3:2 resonance are very similar to those for the 2:1 resonance.

The analysis of the dynamics of these systems was limited to the search of symmetrical librations of the angles σ_1 and σ_2, that is oscillations of these angles around 0 or π. Some numerical experiments have, however, shown that asymmetric librations as found by Beaugé (1994), in which the angles σ_i oscillate around angles not commensurable with π also exist.

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