Two trends in the development of numerical algorithms of celestial mechanics

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The motion of a material particle with the mass m in the gravitational field of the central body with the mass M under the action of conservative forces F_i with the potential function V_i and non–conservative forces P in a rectangular coordinate frame related to the central body M, can be described by equations

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = -\frac{\partial V}{\partial x} + F, \tag{1}$$

with initial conditions $x_0 = x(t_0), \dot{x}_0 = \dot{x}(t_0)$. Here $x = (x_1, x_2, x_3)^T$ is the position vector, t is physical time, $r = |x|$, $\mu = k^2(M + m)$, k^2 is the universal gravitational constant, $V = V(x,t)$, is a perturbed function of potential forces and F is the vector of the acceleration due to the forces which have no potential.

As is well known, Equations (1) are singular in vicinities of the central and perturbing masses. In process of numerical integration these non–uniformities require a regular change of the size of the integration step. It involves the loss of accuracy of numerical solution and wasteful expenses of computer time.

Besides, the solutions of the equations (1) are instable in Lyapunov sense even in the case of the unperturbed motion. This instability intensifies the influence of truncation and round off errors in the process of numerical integration.

The first trend in the development of numerical algorithms of celestial mechanics is related with the construction of transformations enabling completely or partly to avoid the singularity mentioned above.

Initial conditions $x_0 = x(t_0), \dot{x}_0 = \dot{x}(t_0)$ of Equations (1) are determined by the region of possible motion R_0.

In classical way, under the assumption that the law of distribution of errors of observations is close to the normal one, the initial regions of possible motion R_0 are dedermined by the LSM-evaluations of vector of initial parameters $q_0 = \{x_0 = x(t_0), \dot{x}_0 = \dot{x}(t_0)\}$ and by the covariance matrix of its errors $\hat{D}_0$,

$$R_0\colon N\left(\hat{q}_0, k^2\hat{D}_0\right), \quad k = 1, 2, 3,$$

where k is the gain factor of the LSM-evaluations of the covariance matrix of errors in initial parameters.

In the case, when the law of distribution of errors of observations greatly differs from the normal one, one has to search other ways to assign initial domains of the object motion.

In any case the orbital evolution of celestial body should be considered as evolution of the domain of the body possible motion.

Construction of the algorithms for determining the evolution of the possible motion domains represents the second trend in the development of numericsl algorithms of celestial mechanics.

This paper presents a brief summary of the results in the development of both trends obtained by the authors during last several years.

We discuss new Encke-type algorithms in regularising and stabilising variables. The algorithms do not contain the equations for fast variables and display high efficiency in numerical simulating the motion of special asteroids and planetary satellites.

The problem of numerical investigation of close encounters of small bodies with large planets is analysed.

New algorithms for determining initial domains of possible motions are considered. The analysis of using linear and non–linear algorithms for determining evolutions of the domains of possible motions are given. Several interesting numerical examples are given. These examples show that the main merit of the nonlinear method is the fact that evaluations obtained on its basis are much more profound and give a greater amount of interesting information of motion.

New results and the results that have been partially published in [1] are presented.

References

1. Bordovitsyna T. V., Avdyushev V. A., Chernitsov A. M. New Trends in Numerical Simulation of the Motion of Small Bodies of the Solar System. Cel. Mech. and Dyn. Astr., 2001, **80**, 227–247.