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Rotation theory of celestial bodies: dynamical relations of the pole motion and nutation

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1. Problem

By construction Earth rotation theory the studies of the precession–nutation motion and pole motion are considered usually separately. The reason of this approach is obvious. The angle Q between the polar axis of inertia and axis of rotation is small (of order 10^{-6}). The pole perturbations (of the different nature) are very small to give big contribution to the nutation. But in any case we must give evaluations of these possible correlation between pole motion, diurnal rotation and space axis motion (precession and nutation). On the other hand, we know about very high requirements to accuracy of the Earth rotation theory ($10^{-5} - 10^{-6}$ arcsec). If we take into account particularities of the pole motion by construction of the nutation theory the analytical problem will be more complicated as compared with classical approach. To study the above–mentioned effects in the Earth rotation we use some new approaches to construct the Earth rotation theory.

2. Force function. Unperturbed motion

The problem of constructing the analytical theory of the rotational motion of the Earth takes one of the central places in celestial mechanics. Starting from classical papers on this subject the equations of rotational motion in the osculating elements similar to that of Andoyer and angle-action variables have been used (Laplace,1825; Poisson, 1827; Pontecoulant,1829; Tisserand,1891; Andoyer, 1923, and so on). But in constructing the force function of the problem in these variables the authors used usually the approximate developments with additional assumption about small eccentricities of the ellipsoid of inertia. Analytical solution of the problem was of restricted character. In (Kinoshita, 1977) in constructing the Earth rotation theory similar simplifications were used (only the constant of precession was presented in the explicit form in term of elliptic integrals).

We have developed a more general theory to study the rotation of the deformable celestial bodies. The new effective forms of the canonical and non-canonical equations in Andoyer and angle-action variables were suggested. These variables were introduced on the basis of the integrable Euler-Chandler problem for the attitude motion of an elastic body deformed by its own rotation. The unperturbed motion with the elastic property taken into account is reduced to the classical Euler-Poinsot problem with a specially changed principal moment of inertia. The properties of the Euler-Chandler unperturbed motion of the Earth are described in details (Chandler period, eccentricity of the pole trajectory, non-uniform pole motion, etc.). The peculiarities of the Venus pole motion have been studied as well.

3. Perturbations of the pole motion of the deformable isolated body

We have studied the problem of rotation of the isolated deformable body with a shell changeable in time. The components of the external shell tensor of inertia are given as conditionally–periodic functions of time. The analytical formulas for the secular and periodic perturbations have been obtained in Andoyer, angle-action and classical Euler variables. Generally, these formulas are obtained for arbitrary parameters of the considered unperturbed motion (for example for arbitrary unperturbed value of the angle Q between the axis of the angular moment and the body pole axis). The analytical formulas for the precession constant and the additive terms to the Chandler period and diurnal rotation due to the gravitational attraction of the Moon and the Sun are obtained in terms of elliptic functions and integrals. The perturbation theory of the Earth rotation has been constructed in the elastic Andoyer and angle–action variables. Perturbations of the Earth rotation due to the tidal and non-tidal variations of the Earth tensor of inertia in gravitational field of the Moon and the Sun have been obtained.

4. Perturbations

Complete formulas for the first–order perturbations of the rotational motion of an Earth's satellite moving in the gravitational field under the action of the perturbing body (the Moon and the Sun) in the angle–action variables (for Euler and Euler–Chandler unperturbed motions) have been derived. Amplitudes of all perturbations of the first order were presented in terms of elliptic functions and integrals of the action variables. It means that the analytical theory is applicable for study of the attitude motion of the natural and artificial celestial bodies with arbitrary dynamical structure and for arbitrary unperturbed pole motion in accordance with Euler or Chandler–Euler problems. Perturbations in the pole motion are calculated for the two initial values of the angle between polar axis of inertia and vector of the angular momentum Q = $0.12''$ and Q = $0.14''$ (these values correspond to the real averaged pole trajectory). New nutation terms of small amplitudes have been found. Results presented here are perspective for the construction of the high–accuracy theory of the rotation of the Earth and other celestial bodies with a non–ordinary unperturbed rotation (Venus, asteroids, etc.). This work is supported by the grant of RFBR 02-05-64176 and grant of Spain for 2002–2003 Sabatico year at the Alicante university.

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