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Earth penumbra effects on AES motion taking into account the refraction and the extinction of the light in the atmosphere

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The perturbing acceleration **F** of the Artificial Earth Satellite (AES) due to the solar radiation pressure can be described as follows:

$$\mathbf{F} = f \cdot D \cdot \mathbf{N},$$

where D — the expression for the absolute quantity of the acceleration, $\mathbf{N}$ — the unit vector along the resultant of the radiation pressure forces, f — the shadow function, with $f = 0$ when the satellite is in Earth shadow (umbra); $f = 1$ when the satellite is in sunlight; $0 < f < 1$ when the satellite is in partial shadow (penumbra). The penumbra is postulated as an enlarged near–earth space area in which the solar radiation acting on the satellite surface undergoes the refraction and the extinction in the earth's atmosphere. The shadow function can be described as follows:

$$f = \frac{J_1}{J_0}, \quad J_0 = \iint_s I_0 ds, \quad J_1 = \iint_{s_1} I_s ds,$$

where I_0 — the nominal radiation intensity of the solar disk element ds referred to the unit celestial sphere with the centre in the satellite position, s — the nominal area of the solar disk, I_s — the observed radiation intensity of the solar disk element ds taking into account the extinction of the light in the earth's atmosphere, s_1 — the observed area of the solar disk (or the part of the solar disk eclipsed by the Earth) taking into account its deformations due to the refraction of the light in the earth's atmosphere.

The complete theory of the direct solar radiation pressure perturbations acting on AES orbits was presented by some authors [1]–[3]. We have developed the more simple approach to investigating the problem of the refraction and the extinction of the light in the Earth's atmosphere [4]. The relationship for the calculation of the refraction angle is obtained by assumption that the air density depends exponentially on the height. The relationships for the evaluation of the light extinction are derived taking into account its dependence on the light wave length. The variations of the sun energy flow with the wave length were considered as well [5]. The direct use of these integral formulas needs a lot of computer time. The evident computer time reduction may be reached when the satellite is moving on a near–circular orbit. In this case the computation of the shadow function is most efficiently done by a two–step procedure. In step 1, the table of the shadow function values with the angle distances d between the solar disk and the earth disk edge may be calculated. This numerical dependence $f = f(d)$ is considered as a starting one for each specific near–circular orbit. Step 2 of the computation is the approximation of function $f = f(d)$ by some elementary function. The latter dependence is used for calculating f-values. We have used this algorithm to study the penumbra effects in the motion of 12–hour satellites "Navstar". The main results obtained in this paper can be summarized as follows: (1) the effect of displacement of unit vector $\mathbf{N}$ from the solar–satellite direction is too small [6] and can be neglected; (2) the refraction and the extinction of the light in the atmosphere must be taken into account for estimating the shadow function when the trajectory measurements have the accuracy of several centimetres.

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