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Generalized Nordtvedt effect and tests of equivalence principle for rotating bodies

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The most recent results of the lunar laser ranging (LLR) data analysis [1] gives restriction on Einstein Equivalence Principle (EEP) violation due to Nordtvedt effect (proposed in [2]) at the level $\eta_{LLR} = (-0.7 \pm 1.5) \times 10^{-13}$, where $\eta = m_G/m_I - 1$. It means that gravitational binding energy ($\eta_{bind} = E_{bind}/Mc^2 = 4.63 \times 10^{-10}$ for the Earth) gives the same contribution to both inertial and gravitational masses with accuracy about 0.03%.

Here it is proposed a new application of LLR results for testing EEP violation due to rotational energy of the Earth which corresponds to $\eta_{rot} = E_{rot}/Mc^2 = 3.97 \times 10^{-13}$. In general relativity (GR) according to EEP a rotating body falls down as non-rotating one (up to the tidal effects, which may be neglected at present accuracy). To study theoretical predictions for possible violation of EEP the Thirring-Feynman field gravity theory (FGT) is considered [3], [4]. It is based on the principle of universality of gravitational interaction (UGI) which states that the interaction Lagrangian has the form $\Lambda_{(int)} = -\frac{1}{c^2}\psi_{ik}T^{ik}$ where ψ_{ik} is the tensor potential of gravity field and T^{ik} is the energy momentum tensor of any kind of fields. It means that UGI states that gravity is universal force in the sense that it "sees" any material field through its energy momentum tensor (including gravity field itself).

Using Lagrangian formalism of the relativistic field theory the post-Newtonian equation of translational motion for rotating bodies is derived in the frame of FGT. The PN-3-acceleration of the rotating body in FGT has the form:

$$\frac{d\vec{V}}{dt} = -\Big(1 + \frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} + \frac{I\omega^2}{Mc^2}\Big)\vec{\nabla}\varphi_N + 4\frac{\vec{V}}{c}\Big(\frac{\vec{V}}{c}\cdot\vec{\nabla}\varphi_N\Big)$$

$$+\frac{3}{Mc^2}\int [\vec{\omega}\vec{r}]([\vec{\omega}\vec{r}]\cdot\vec{\nabla}\varphi_N)dm$$

Following the method by Landau & Lifshitz ([5],p.331) for a post–Newtonian derivation of rotational effects in relativistic gravity, we used for the velocity $\vec{v}$

of an element dm the form $\vec{v} = \vec{V} + [\vec{\omega}\vec{r}]$, where $\vec{V}$ is the translational velocity of the body, $\vec{\omega}$ is the angular velocity, $\vec{r}$ is the radius vector of an element dm relative to its center of inertia, so that $\int \vec{r}dm = 0$, $M = \int dm$ is the total mass of the body, I is its moment of inertia.

The equation of motion of a rotating body shows that the orbital velocity of the center of mass of the body will have additional perturbations due to rotation of the body. The last term in this equation depends on the direction and value of the angular velocity $\vec{\omega}$. It has an order of magnitude v_{rot}^2/c^2 and may be measured in laboratory experiments and astronomical observations. Note that the binding energy does not enter into the equation of motion so that both FGT and GR classical Nordtvedt effect are absent. A generalized Nordtvedt effect includes possible rotational effects derived in FGT from UGI. The additional perturbation terms in the equation of motion of rotating body are determined by the direction and value of angular velocity: $\delta g = (\eta^0 + \eta^1 cos\Omega t)g_N$. Taking into account the inclination of the rotation axis of the Earth on the orbit around the Sun and using the momentum of inertia of the Earth $I = 0.33\ MR^2$ we get for the constant perturbation the value $\eta_{FGT}^0 = +0.32 \times 10^{-13}$, and for the variable part the amplitude $\eta_{FGT}^1 = 0.96 \times 10^{-13}$ with the period $P = 0.5P_{orb}$, where P_{orb} is orbital period of the Earth. These values are very close to the modern accuracy of LLR and show the importance of further improvement of the LLR experiment to test EEP violation for rotating bodies.

References

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