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Estimation of the PPN parameter γ with geodetic VLBI

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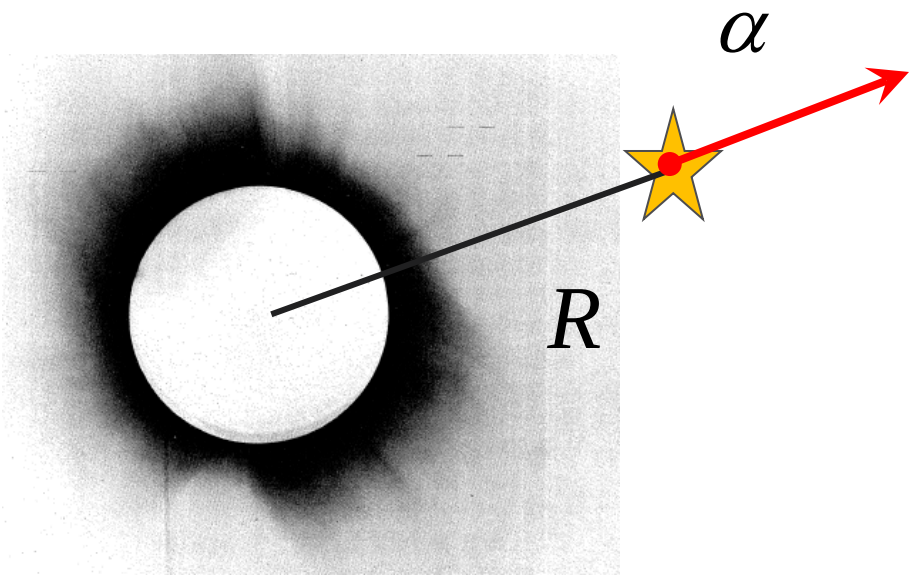
General Relativity

1. Einstein predicted the light deflection due to Solar gravitational field (1915) as much as 1”.75 for grazing light, i.e. R is the Solar radii.

$$\alpha = \frac{4GM}{c^2 R}$$



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Optical experiments (Mikhailov, 1959; Brune, 1976)

Eclipses	*A* as given by observer	*A* as given by new reduction
1919 – Brazil	1″·98	2″·07 ± 0″·09
1922 – Australia	1·72	1·83 ± 0·11
1929 – Sumatra	2·24	1·96 ± 0·08
1936 – USSR, Far East	2·70	2·68 ± 0·37
1947 - Brazil	2·01	2·20 ± 0·38
1952 - Sudan	1·70	1·43 ± 0·18
	Simple mean 2·03 ± 0·10	

1973 – Mauritania 0”.95 +/- 0”.11

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Test of General Relativity

Accurate equation for all ranges of separation angles θ from 0 to 180º

Shapiro (1967), Ward (1970)

$$\alpha = \frac{4GM}{c^2 R}$$

$$\alpha = \frac{2GM}{c^2 r}\frac{1+\cos\theta}{\sin\theta} \qquad \alpha = \frac{2GM}{c^2 r}\frac{\sin\theta}{1-\cos\theta} \qquad \alpha = \frac{2GM}{c^2 r} ctg\frac{\theta}{2}$$

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Test of General Relativity with VLBI

Parametrised **P**ost-**N**ewtonian (PPN) approximation (Will, 1973)

$\gamma \equiv 1$ in General Relativity

$$\alpha = \frac{4GM}{c^2 R} = \frac{2(1+\gamma)GM}{c^2 R}$$

$$\alpha = \frac{2GM}{c^2 r} ctg \frac{\theta}{2} = \frac{(1+\gamma)GM}{c^2 r} ctg \frac{\theta}{2}$$

corresponds ~0”.75 for grazing light

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Test of General Relativity with VLBI

Table 1. Review of γ-determination using geodetic VLBI data.

Author(s)	Year	γ	σ	Data
Counselman *et al.*	1974	0.98	± 0.06	1972 occultation of 3C279 by the Sun
Fomalont & Sramek	1975	1.0075	± 0.022	1974 occultation of 3C279 by the Sun
Fomalont & Sramek	1976	1.0035	± 0.018	1974 and 1975 occultation of 3C279 by the Sun
Robertson & Carter	1984	1.008	± 0.005	MERIT, POLARIS, IRIS
Carter, Robertson & MacKay	1985	1.000	± 0.003	POLARIS, IRIS since 1980
Robertson, Carter & Dillinger	1991	1.000	± 0.002	POLARIS, IRIS, CDP since 1980
Lebach *et al.*	1995	0.9996	± 0.0017	1987 occultation of 3C279 by the Sun
Eubanks *et al.*	1997	0.99994	± 0.00031	Geodetic VLBI sessions from 1979 to 1997
Shapiro *et al.*	2004	0.9998	± 0.0002[1]	Geodetic VLBI sessions from 1979 to 1999
Lambert & Le Poncin-Lafitte	2009	0.99984	± 0.00015[2]	Geodetic VLBI sessions from 1979 to 2008

From “Cassini” ~ $2 \cdot 10^{-5}$ Bertotti et al (2003)

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Test of General Relativity with VLBI

1. The current precision of the gamma estimate is 2×10^{-4} , whereas numerous modifications of the theory of gravity predicts deviation at level of $10^{-6} \div 10^{-7}$
2. VLBI observations at all separation angles are used.
3. The light deflection may be linked to the gravitational delay.

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Time delay vs light deflection

$$\tau_{geom} = \frac{-\frac{(\boldsymbol{b}\cdot\boldsymbol{s})}{c}\left(1-\frac{(1+\gamma)GM}{c^2 r}-\frac{\left\langle \boldsymbol{V}_{\oplus}^{\,2}\right\rangle}{2c^2}-\frac{(\boldsymbol{V}_{\oplus}\cdot\boldsymbol{w}_2)}{c^2}\right)-\frac{1}{c^2}(\boldsymbol{b}\cdot\boldsymbol{V}_{\oplus})\left(1+\frac{(\boldsymbol{V}_{\oplus}\cdot\boldsymbol{s})}{2c}\right)}{1+\frac{1}{c}(\boldsymbol{s}\cdot(\boldsymbol{V}_{\oplus}+\boldsymbol{w}_2))}$$

$$\tau_{grav} = \frac{(1+\gamma)GM}{c^3}\ln\frac{|\boldsymbol{r_1}|+(\boldsymbol{r_1}\cdot\boldsymbol{s})}{|\boldsymbol{r_2}|+(\boldsymbol{r_2}\cdot\boldsymbol{s})},$$

$$\tau_{GR} = \tau_{grav} + \tau_{coord} = \frac{(1+\gamma)GM}{c^3}\ln\frac{|\boldsymbol{r_1}|+(\boldsymbol{r_1}\cdot\boldsymbol{s})}{|\boldsymbol{r_2}|+(\boldsymbol{r_2}\cdot\boldsymbol{s})} + \frac{(1+\gamma)(\boldsymbol{b}\cdot\boldsymbol{s})}{c}\frac{GM}{c^2 r}$$

The second term in time delay is important!

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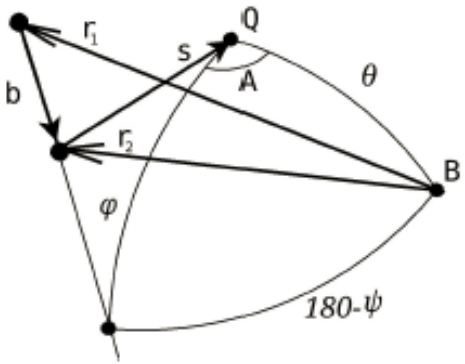
Difference between two models



$$\tau_{GR} = \frac{2GMb}{c^3 r}\frac{\sin\varphi\sin\theta\cos A}{1-\cos\theta} +$$

$$+\frac{GM}{c^3}\frac{b^2(1-\cos^2\varphi\cos^2\theta)}{r^2(1-\cos\theta)} - \frac{GM}{c^3}\frac{b^2\sin^2\varphi\sin^2\theta\cos^2 A}{r^2(1-\cos\theta)^2}$$

$$\tau_{GR} \sim \frac{2GMb}{c^3 r}\frac{\sin\varphi\sin\theta\cos A}{1-\cos\theta} = \alpha\frac{b}{c}\sin\varphi\cos A$$

$$\alpha = \frac{2GM}{c^2 r}\frac{\sin\theta}{1-\cos\theta}$$

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Small-angle approximation

$$\tau_{GR} = \frac{2GMb}{c^3 r}\frac{\sin\varphi\sin\theta\cos A}{1-\cos\theta} + \frac{GM}{c^3}\frac{b^2(1-\cos^2\varphi\cos^2\theta)}{r^2(1-\cos\theta)} - \frac{GM}{c^3}\frac{b^2\sin^2\varphi\sin^2\theta\cos^2 A}{r^2(1-\cos\theta)^2}$$

$$\tau_{GR} = \frac{4GM}{c^2 R}\frac{b}{c}\sin\varphi\cos A - \frac{2GM}{c^2 R}\frac{b}{R}\frac{b}{c}\sin^2\varphi\cos 2A$$

$$\tau_{GR} = \alpha\frac{b}{c}\sin\varphi\cos A + \ldots$$

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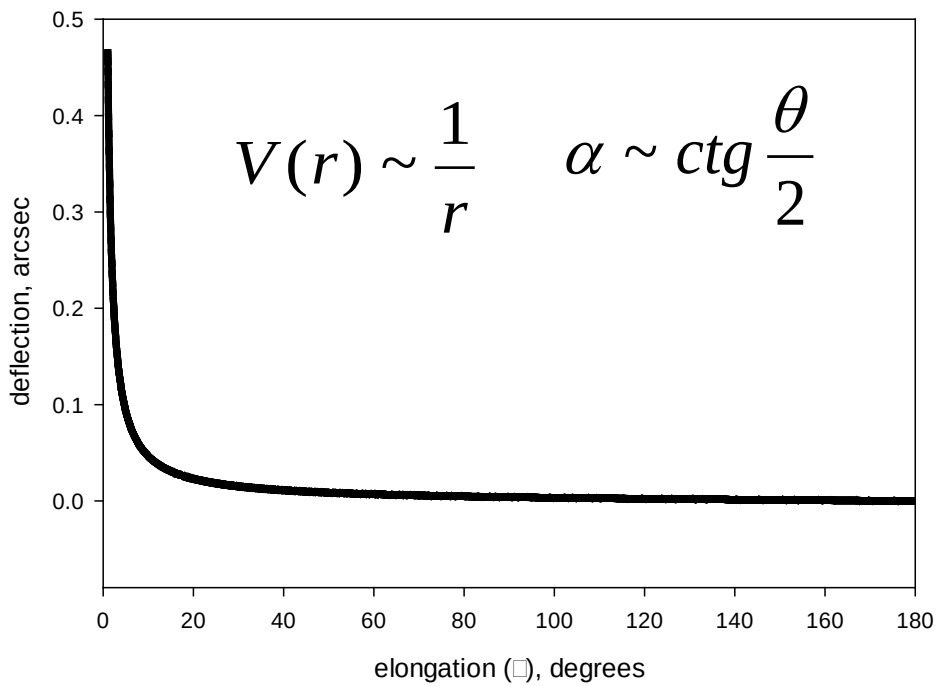
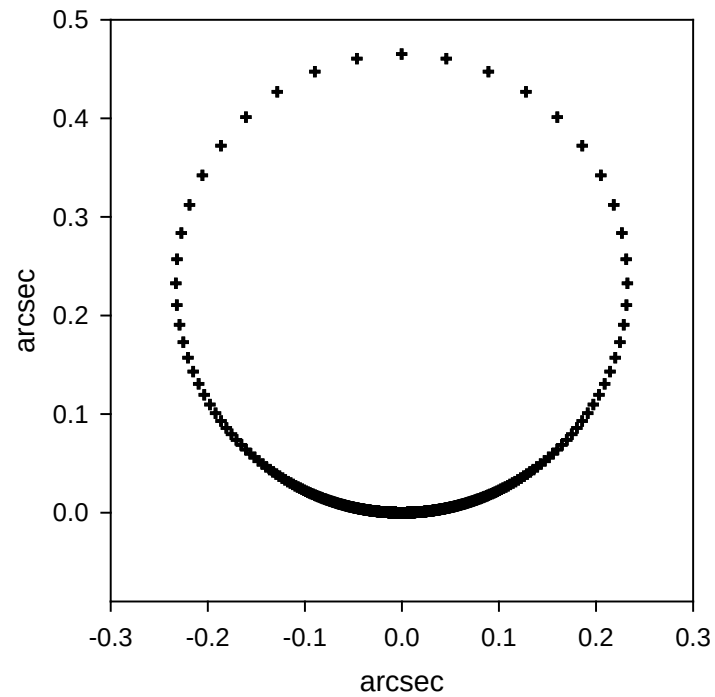
Light deflection in VLBI

$$\alpha = \frac{4GM}{c^2 R}$$ at the small angle approximation, Einstein (1915)

$$\alpha = \frac{2GM}{c^2 R}\frac{\sin\theta}{1-\cos\theta}$$ Shapiro, Science, (1967); Ward, ApJ, (1970)





For a radio source within 1° from Sun

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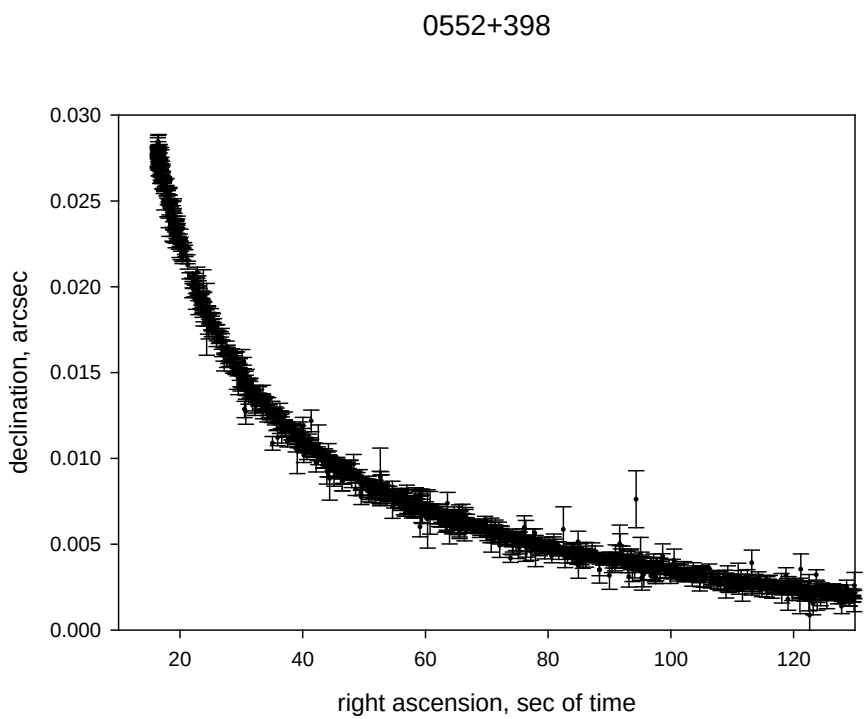
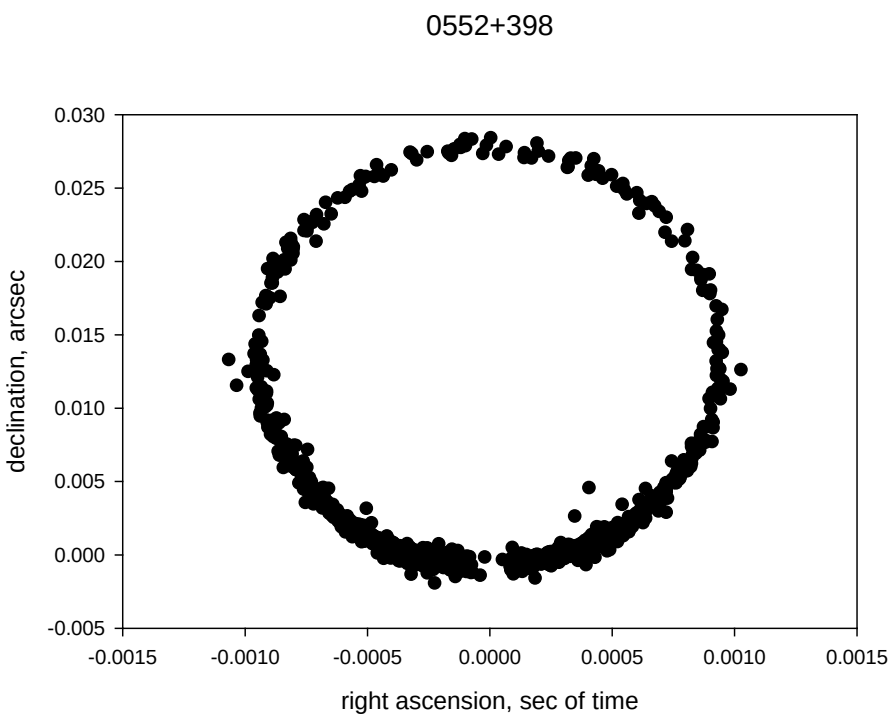
Light deflection angle 0552+398

$$\alpha = \frac{2GM}{c^2 R} \frac{\sin\theta}{1-\cos\theta}$$





Real data, 1991-2001, each session, each radio source

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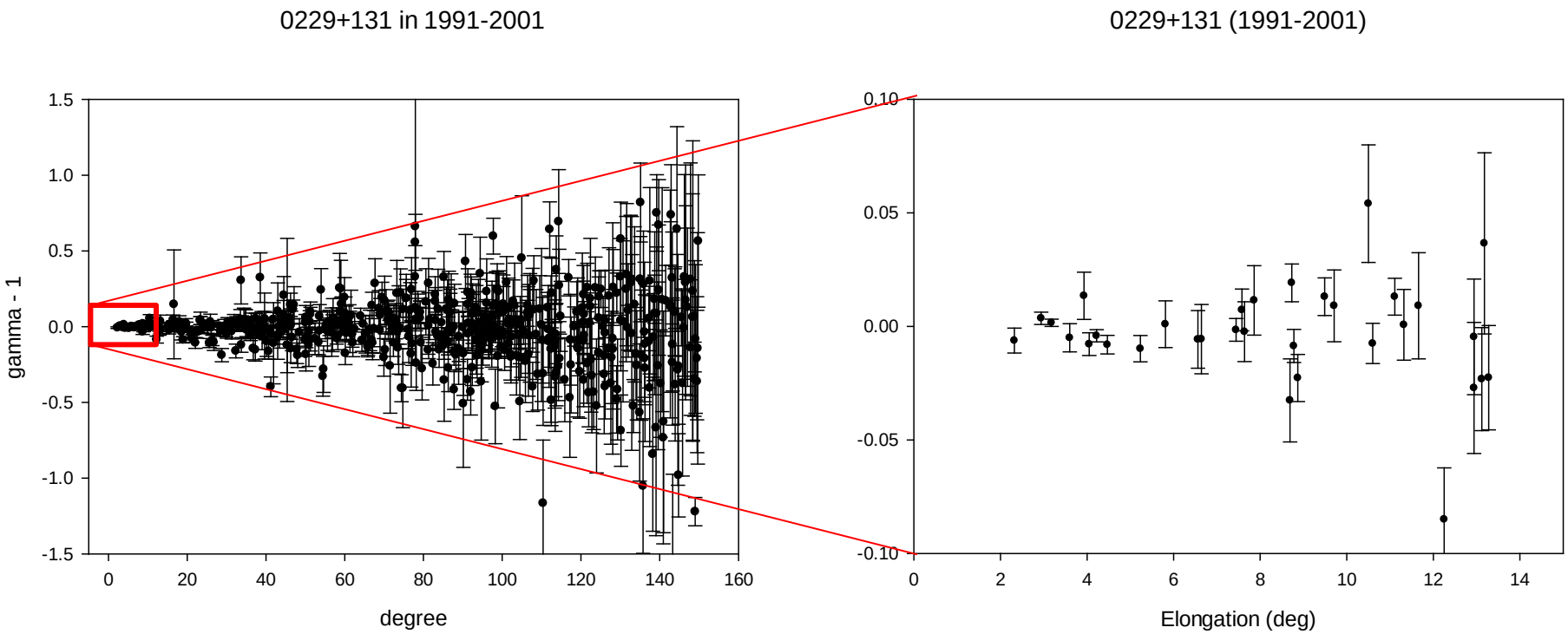
Parameter γ - VLBI is better at small angles



$$\alpha = \frac{(\gamma+1)GM}{c^2 r}\frac{\sin\theta}{1-\cos\theta}$$

$$\gamma = \alpha\frac{c^2 r}{GM}\frac{1-\cos\theta}{\sin\theta} - 1$$

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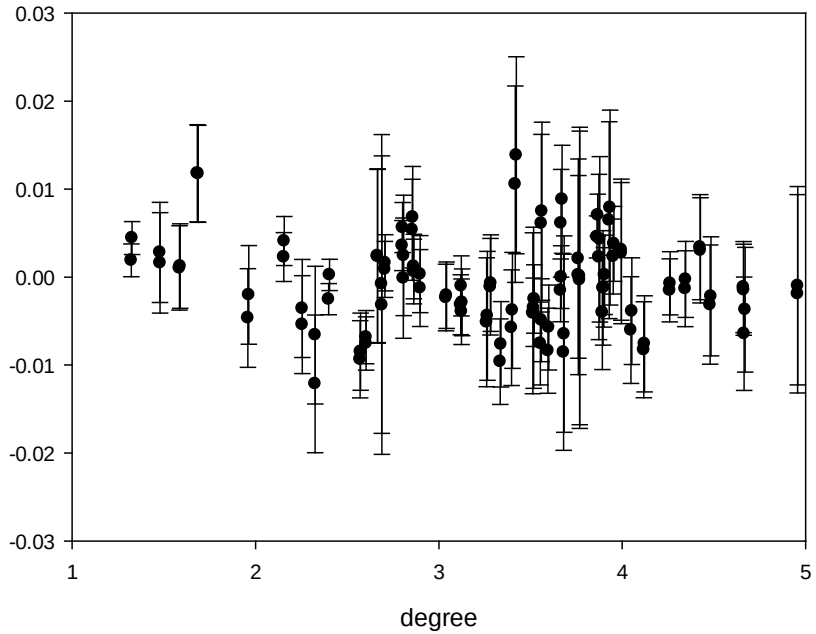
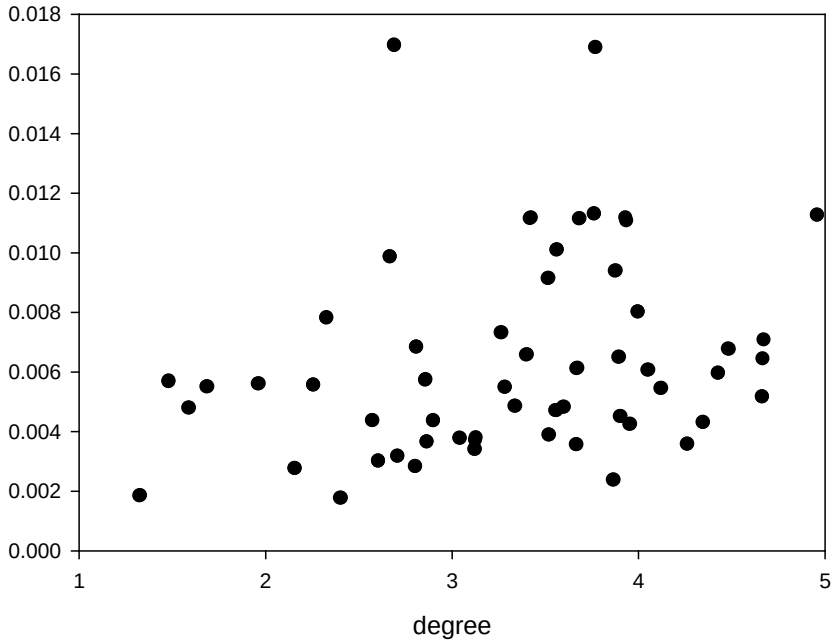
VLBI is better at small angles

60 sessions between 1993 and 2015; 1662 single S/X delays

$$\Delta\gamma = (-1.2 \pm 6.2) \times 10^{-4}$$



Correction to γ=1



Parameter γ formal error

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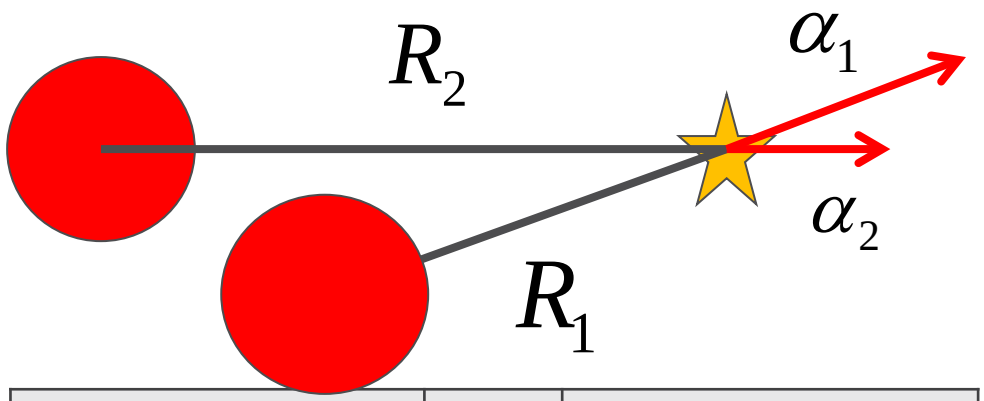
Secondary deflection angle (two telescopes)

$$\Delta\theta = \theta_2 - \theta_1 \sim b/r \sim 8''$$

$$\alpha_1 = \frac{4GM}{c^2 R_1} \qquad \alpha_2 = \frac{4GM}{c^2 R_2}$$

$$\alpha'' = \alpha_2 - \alpha_1 \sim \frac{4GMb}{c^2 R^2}$$



R (km)	α	α'', b=10000 km
700,000 (grazing light, 0°.25)	1".75	0".0125
$2.8 \cdot 10^6$ (1°)	0".44	0".00080
$5.6 \cdot 10^6$ (2°)	0".22	0".00020
$15 \cdot 10^6$ (5°)	0".09	0".00003

Depends on baseline length!

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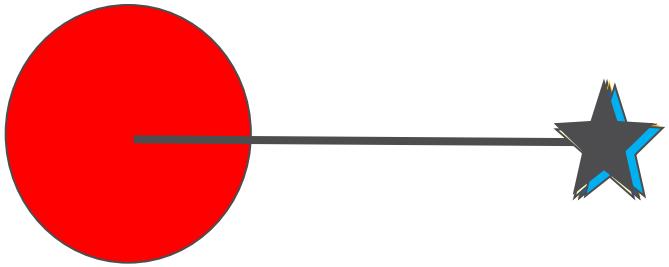
Secondary deflection angle (two telescopes)

$$\tau_{GR} = \frac{4GM}{c^2 R}\frac{b}{c}\sin\varphi\cos A - \frac{2GM}{c^2 R}\frac{b}{R}\frac{b}{c}\sin^2\varphi\cos 2A$$

$$\alpha_1 = \frac{4GM}{c^2 R_1} \qquad \alpha_2 = \frac{4GM}{c^2 R_2}$$

$$\alpha'' = \alpha_2 - \alpha_1 \sim \frac{2GMb}{c^2 R^2}$$



$$\alpha'' = \frac{2GMb}{c^2 R^2}\frac{\sin\varphi\cos 2A}{\cos A}$$

Not only the delay but the deflection angle depends on the baseline length!

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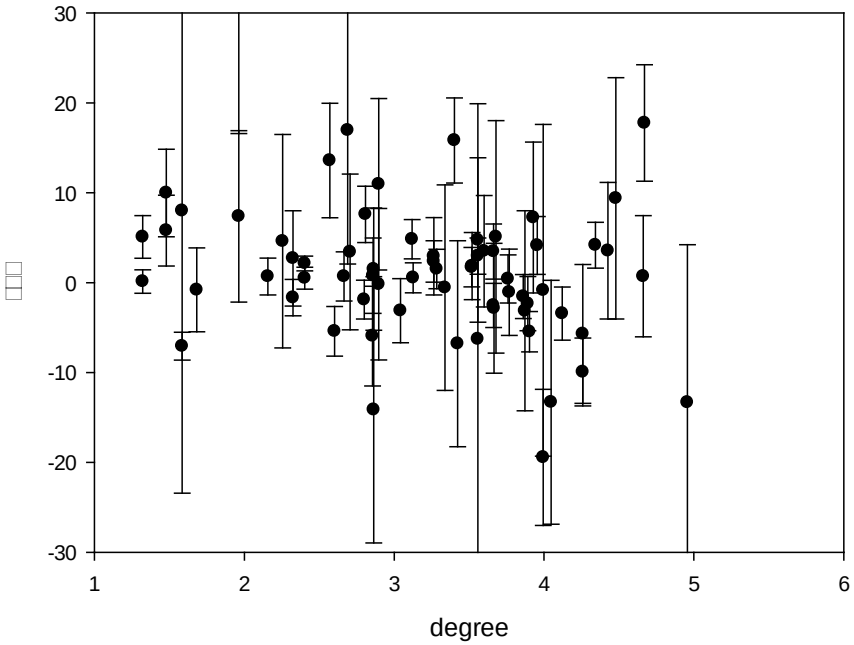
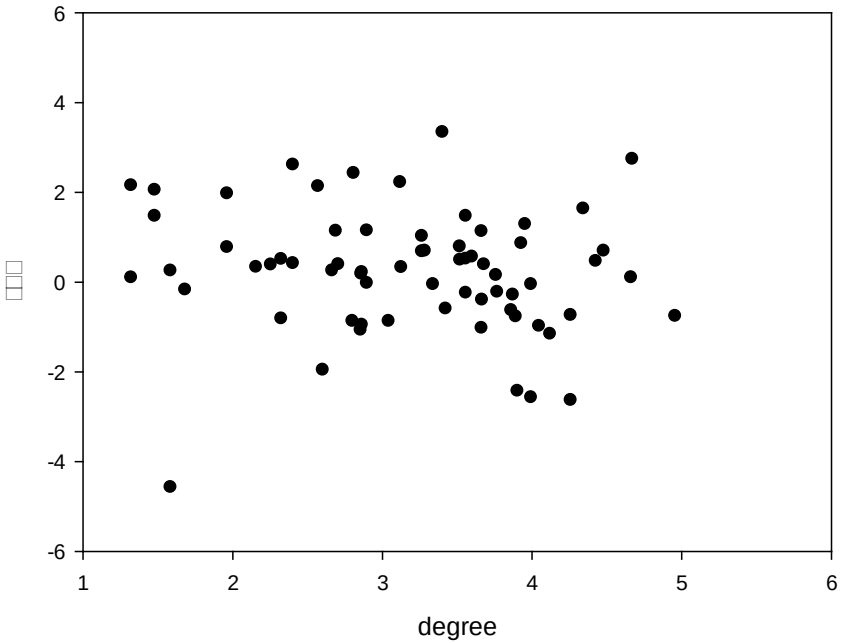
Secondary deflection angle

$$\tau = \frac{(1+\gamma)GMb}{c^2R^2}\sin^2\varphi\cos 2A$$

γ



$\frac{\gamma}{\sigma_\gamma}$



$$\gamma = 1.79 \pm 0.36$$

$$\gamma = 2.24 \pm 0.36$$

Without single outlier (21-Oct-1997; 1334-127)

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Conclusion

1. "Main" deflection angle is estimated with accuracy of 10^{-4} from a global set of VLBI data whereas a limited set of data provides accuracy of $\sigma_\gamma \sim 6\times10^{-4}$
2. For the secondary effect the light deflection angle looks larger than expected ($\gamma = 2.24 \pm 0.36$). More observations at separation less than 2º are required to get a better statistic.
3. Source structure effect may bias the parameter estimate – needs to be investigated
4. Cosmological factor?

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Any Questions?

Thank you for your attention

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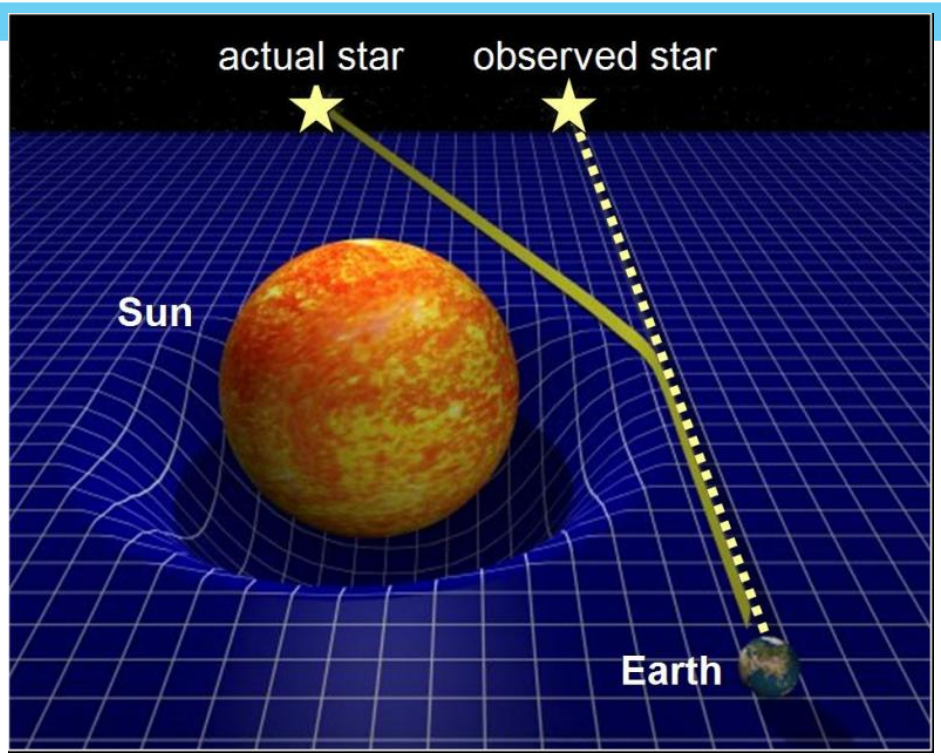
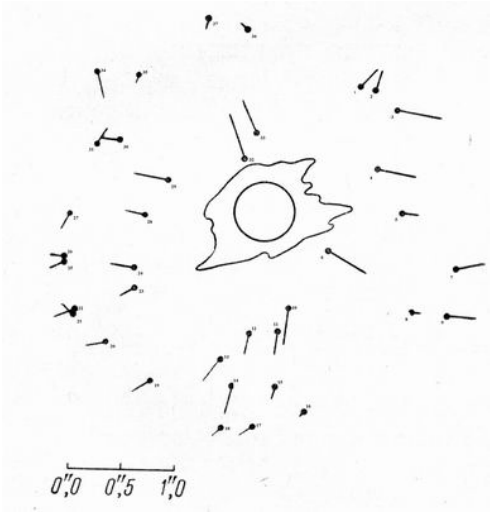
General relativity – light bending



Special expeditions to observe total Solar eclipses since 1919

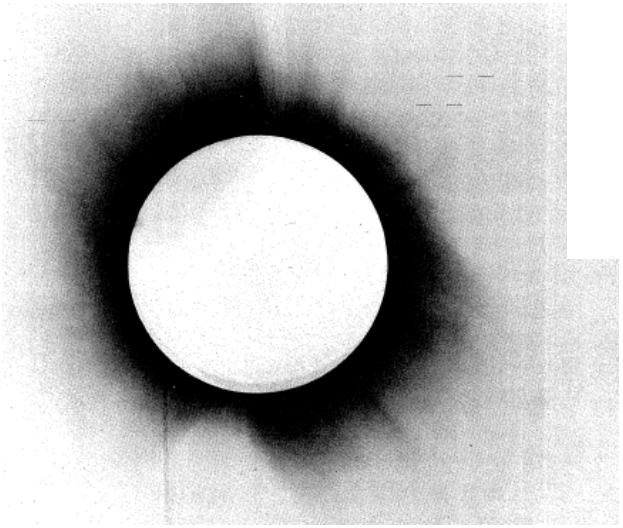


Now VLBI is doing it every session!

Test of General Relativity

The major problem is the Einstein's equation is the small angle approximation. Inappropriate at larger separation from the Sun

$$\alpha = \frac{4GM}{c^2 R}$$



$$\Delta r = \frac{A}{r} + Br,$$

A – light deflection

B – photoplate scale

Test of General Relativity (1919)

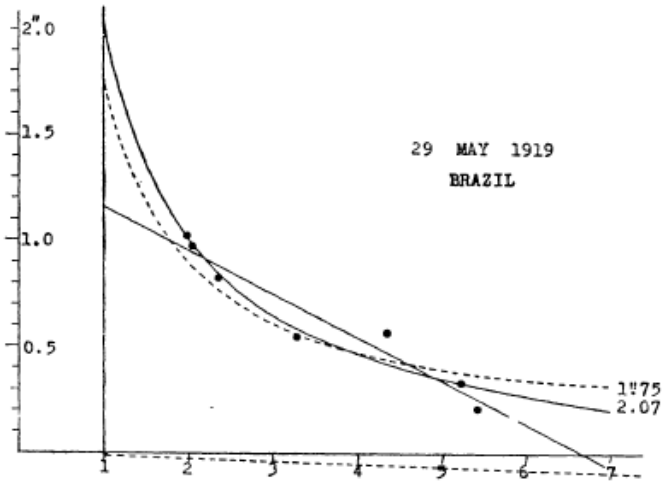


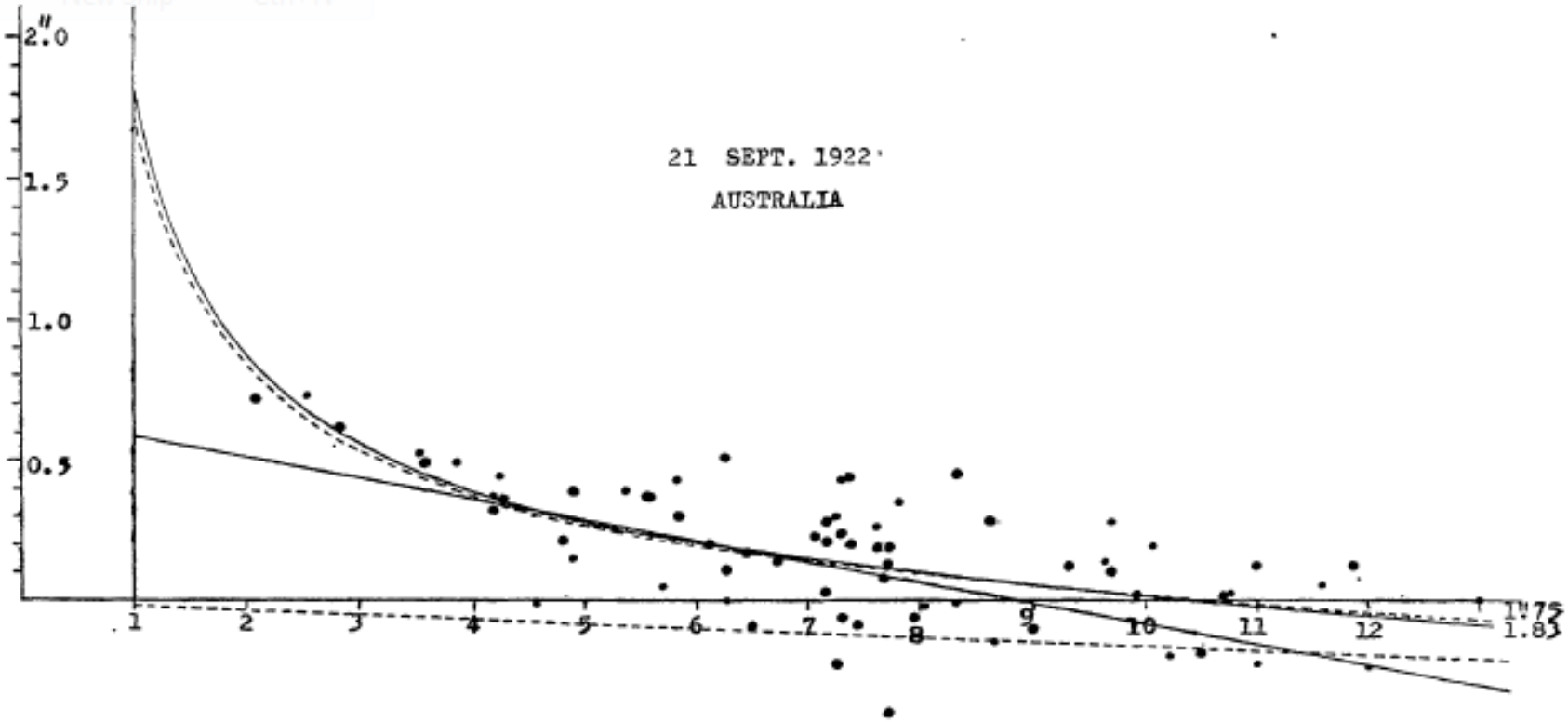
FIG. 1

$$\Delta r = \frac{A}{r} + Br,$$

A – light deflection

B – photoplate scale

Test of General Relativity (1922)



Shapiro (1967)

The theoretical expression for the bending can be obtained, for example, by calculating in isotropic coordinates (5) the difference η in angle of arrival between a ray propagating rectilinearly and according to general relativity. To first order in r_0, we find

$$\eta = \frac{2r_0}{r_e} \tan\left(\frac{\theta}{2}\right) \qquad (1)$$

where $r_0 \approx 1.5$ km denotes the so-called gravitational radius of the sun (6), r_e the heliocentric coordinate distance of the earth observer at the time of detection, and θ the planet-sun-observer angle with the planet's position being at the time of reflection.

$$\alpha = \frac{2GM}{c^2 r} ctg \frac{\theta}{2}$$

Here only terms linear in M^*/r_0 have been retained in the numerator and denominator. We can now determine $\delta\theta_s = \theta_s - \theta'_s$. Since this number is very small,

$$\delta\theta_s \sim \tan(\theta_s - \theta'_s) \equiv \frac{\tan\theta_s - \tan\theta'_s}{1 + \tan\theta_s \tan\theta'_s}. \tag{A9}$$

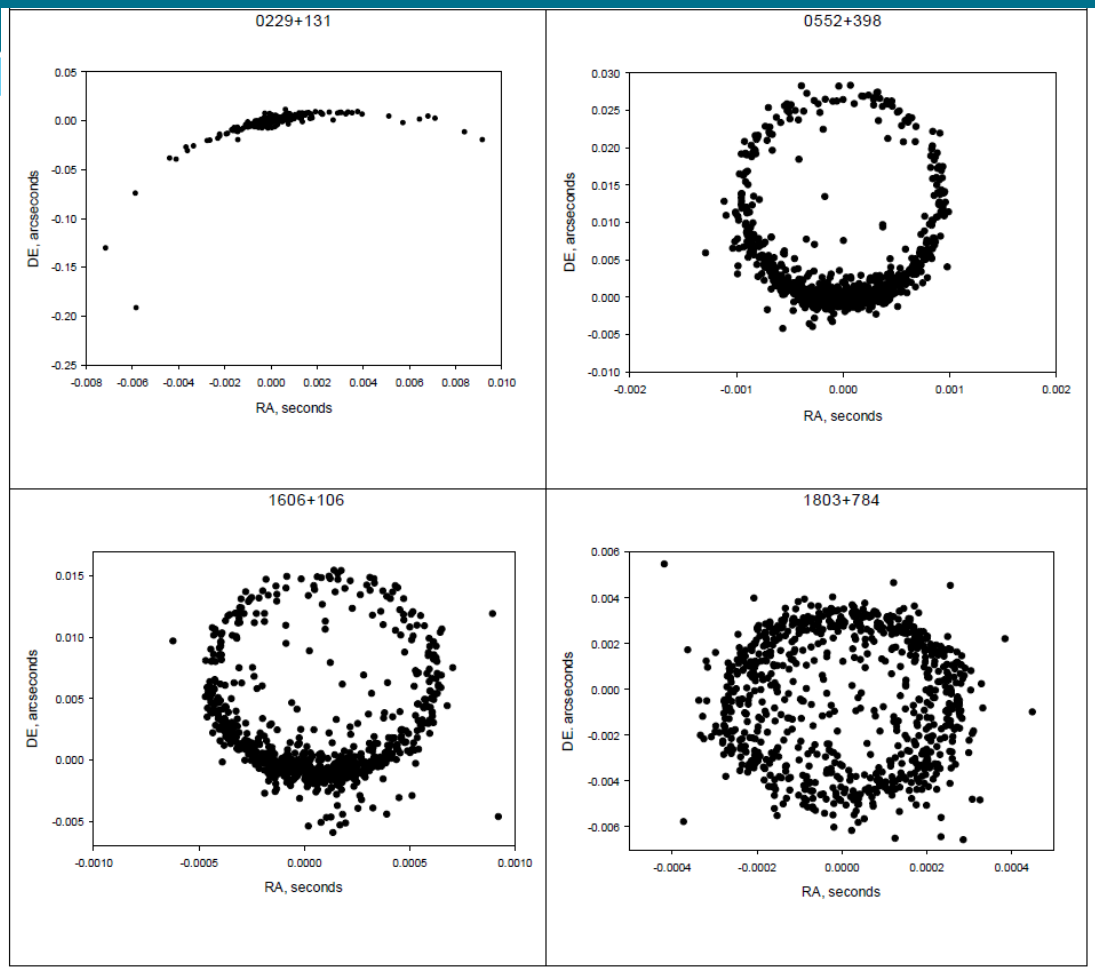
A straightforward calculation by using equation (A8) in equation (A9) yields, to first order in M^*/r_0,

$$\delta\theta_s = (2M^*/r_0)\left(\frac{1+\cos\theta'_s}{\sin\theta'_s}\right) = \frac{4GM}{c^2 l}\frac{(1+\cos\theta_s)}{2} + O\left[\left(\frac{GM}{c^2 l}\right)^2\right]. \tag{A10}$$

REFERENCES

Adler, R., Bazin, M., and Schiffer, M. 1965, *Introduction to General Relativity* (New York: McGraw-Hill Book Co.).

$$\alpha = \frac{2GM}{c^2 r}\frac{1+\cos\theta}{\sin\theta}$$



$$\alpha = \frac{2GM}{c^2 r} \frac{\sin\theta}{1-\cos\theta}$$

Secondary deflection angle

$$\alpha'_2 = \frac{4GM}{c^2 R_2}$$

$$\alpha_2 = \alpha'_2\left(1 - \frac{b}{2R}\frac{\sin\varphi\cos 2A}{\cos A}\right) = \alpha'_2 + \alpha''_2$$

Table 1. Einstein deflection angle and the secondary deflection angle for the Sun.

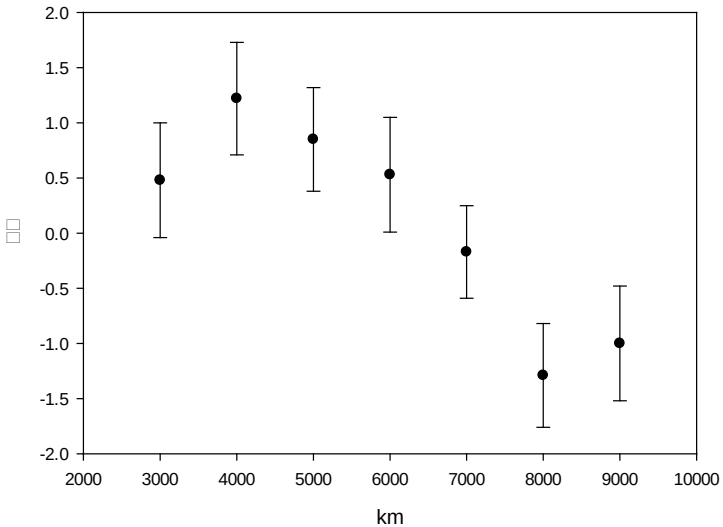
R_2, km	θ	α'_2	α''_2, mas	
			b = 3,000 km	b = 10,000 km
700,000	0°.25	1″.75	3.75	12.5
2,800,000	1°.0	0″.44	0.23	0.80
5,600,000	2°.0	0″.22	0.06	0.20
14,000,000	5°.0	0″.09	0.01	0.03

Latitude deflection – source structure effect?

$$\Delta\gamma = (-1.2 \pm 6.2) \times 10^{-4}$$

1662 single S/X delays

Baseline length (km)	Estimate of Δγ with formal error x10^(-4)	Number of observations
1000 – 5000	4.8 +/- 5.2	498
2000 – 6000	12.2 +/- 5.1	662
3000 – 7000	8.5 +/- 4.7	937
4000 – 8000	5.3 +/- 5.2	1152
5000 – 9000	-1.7 +/- 4.2	902
6000 – 10.000	-12.9 +/- 4.7	791
7000 – 11.000	-10.0 +/- 5.2	381



$$\Delta\gamma = (-1.7 \pm 4.2) \times 10^{-4}$$

902 single S/X delays at 5000-9000 km