Modelling of lightcurves of minor planetary satellites

A. V. Melnikov

Pulkovo Observatory, St. Petersburg, Russia

This report presents results of modelling of lightcurves of minor planetary satellites. Lightcurves of two satellites of Saturn, namely Hyperion (S7) and Phoebe (S9), were modeled using algorithms and programs developed for calculating the rotational dynamics and constructing theoretical lightcurves of planetary satellites. The modelling was based on Pulkovo sets of observations carried out by Devyatkin et al. [1]. Hyperion's observations performed earlier by Klavetter [2] were also used for modelling.

The following assumptions were made: the satellite is a nonspherical tri-axial rigid body; the planet is considered to be a gravitating point. In case of Hyperion the motion in the perturbed elliptic orbit was considered, because its orbit is subject to strong short-period perturbations from Titan. The orbit of Phoebe was taken to be a fixed ellipse, and mean values of the orbital elements [3] were used. In case of Phoebe the perturbations are essentially smaller; besides, the time interval for the modelling is less than the orbital period.

In calculations of the observed stellar magnitude it was assumed that a satellite is a tri-axial ellipsoid, and its surface is orthotropic: the light flux from the satellite is proportional to the area of projection of the visible illuminated part of the satellite's surface on the celestial sphere. Deviations from orthotropicity for the reflecting surface were taken into account by means of correction of the model lightcurves for the "Sun – satellite – observer" phase angle using a definite phase function.

The problem of fitting an observed lightcurve with the model one is solved by varying the initial data and values of the parameters of the problem. As an initial step a rough approximation to the observed lightcurve was found by minimizing the sum of squares of deviations of theoretical values of the satellite's stellar magnitude from the observed ones. Finally, the initial data and values of the parameters were refined by the steepest descent (gradient) method.

The rotational states of satellites and values of the parameters of phase function were deduced in this way for four sets of observational data on Hyperion (three sets of Pulkovo observations carried out by Devyatkin et al. [1] and a set of observations carried out by Klavetter [2]) and for a set of Pulkovo observational data [1] on Phoebe.

In case of Hyperion, the initial data, which specify the model curve best fitting the observed one, are situated in a chaotic component of phase space of the rotational motion. The computation of the maximum Lyapunov characteristic exponent (MLCE) was performed for all sets of the initial data and values of the parameters of the problem. The computed values of the MLCE are close to its analytical estimates calculated by means of the separatrix map theory [4]. For all the sets, the value of the MLCE is greater then zero. The distinction of the value of the MLCE from zero is another indicator of the chaotic character of the rotation. One can make conclusion that Hyperion in the period covered by observations was in the chaotic regime of the rotational motion.

The initial data for trajectories of the rotational motion of Phoebe are situated in the regular domain of phase space of the rotational motion. The value of the MLCE calculated for the deduced initial data and values of the parameters is close to zero. The zero value of the MLCE again points on the regular character of the rotation. The obtained period of rotation of Phoebe is equal to $9.3^{\rm h}$. This is close to an earlier estimation by Kruse et al. [5].

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