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Numerical stabilization of orbital motion

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As it is well-known, Lyapunov's instability of differential equations when being integrated numerically considerably extends various errors inevitably accompanying any numerical process. Errors in an integrating step appear as errors in initial data for the next step and are increased by the instability step by step together with other errors originating later. Therefore, the stable equations are more preferable for numerical integration.

In this work the author examines mainly the stabilization in some interesting problems of orbital dynamics focusing on popular Baumgarte's method [1] and its applications in satellite, asteroid and stellar problems.

It is shown that in phase space ($\mathbf{z}$) of coordinates and their derivatives, small discrepancy $\Delta\mathbf{z}$ between two near elliptical orbits increases in time with the speed proportional to the discrepancy ΔH_K between the Keplerian energies. This gives arguments for the stabilization by H_K in the elliptical two–body problem.

The author studies theoretical and numerical aspects of Baumgarte's stabilization applied to the differential equations of orbital dynamics in the two– and three–body problems. A simple approach is proposed for stabilizing the equations of almost circular motion ($e < 0.001$). Nacozy's method [2] and other techniques are briefly outlined and only their ideas are exposed.

By using Baumgarte's technique, the author derives stabilized equations for the perturbed restricted three–body problem. These equations as well as the stabilized equations, obtained by Baumgarte for the perturbed Keplerian problem [1], are used for simulating the orbits of specific celestial objects: Amalthea, Thebe, Io, Europa, Ganymede and Callisto (close Jovian satellites); Alinda (asteroid) and some outer planet of a hypothetical binary star.

Numerical results show that stabilization can raise the accuracy of numerical integration by several orders. Its exceptional abilities reveal in the satellite problems. In particular, in the case of Amalthea the stabilization is able to raise the accuracy of integration almost by 5 orders (Fig. 1). In principle, the non–stabilized simulations can attain the same accuracy but at the expense of computing time.

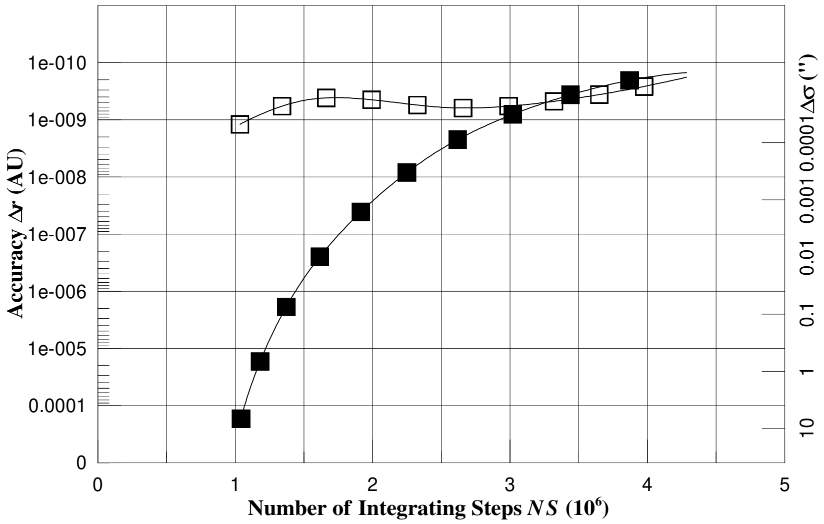

Fig. 1. Accuracy–speed characteristics for stabilized (□) and non–stabilized (■) simulations. Amalthea: 100 years. The accuracy in arc relative to the Earth is given at the right scale. Here Δr is the calculation accuracy for the position vector evaluated by the well–known forward–and–backward integration method, and NS is a number of integration steps over a whole interval of integration. The results have been obtained by 15-order Everhart's numerical integration method.

From this point of view, stabilization can also be regarded as a tool to raise the speed of simulation.

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References

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2. Nacozy P. E. The Use of Integrals in Numerical Integrations of the N-Body Problem, Astrophys. Space Sci., 1971, **14**, 40–51.